

2026

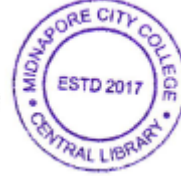
6th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 12-T

(Single Core Major)

(Group Theory-II)



Full Marks : 60

Time : Three Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

Answer any **ten** questions : $2 \times 10 = 20$

1. "If G_1 and G_2 are two finite groups such that $|G_1| > |G_2|$, then $|Aut(G_1)| > |Aut(G_2)|$." — Check whether this statement is true or false. Give reasons.
2. Find all the characteristic subgroups of S_3 .
3. Show that a cyclic group of order 4 cannot be expressed as an internal direct product of two of its subgroups of order 2.
4. Find the order of the element $(\bar{5}, \bar{5})$ in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{16}$.

P.T.O.

(2)

5. Let G be a finite group that has only two conjugacy classes. Then show that $|G|=2$.
- ✓ 6. Using Sylow's theorem prove that any group of order 63 is not simple.
- ✓ 7. Is $\mathbb{Z} \oplus \mathbb{Z}$ a cyclic group? Justify your answer.
- ✓ 8. Find the number of elements of order 10 in the group $\mathbb{Z}_{50} \oplus \mathbb{Z}_{25}$.
9. Show that the kernel of the group action is a subgroup.
10. Give an example of an infinite p -group, p being a prime.
11. Determine $\text{Aut}(Z)$, where Z denotes the infinite cyclic group under addition.
- ✓ 12. Let G_1, G_2 be commutative groups. Show that direct product $G_1 \times G_2$ is a commutative group.
- ✓ 13. Show that any group of order p^2 is commutative, where p is prime.
- ✓ 14. State Cauchy's theorem.
15. Find all Sylow 2-subgroups of A_4 .



Group - B

Answer any *four* questions : $5 \times 4 = 20$

- ✓ 16. Let H and K be normal subgroups of a group G such that $G=HK$ and $H \cap K = \{e\}$. Prove that $G \cong H \times K$.

(3)

17. Prove that the automorphism group of $(\mathbb{Z}_n, +)$ is isomorphic with U_n .
- ✓ 18. Let G be a group acting on a non-empty set S and $x \in S$. Define the stabilizer G_x of x . Then prove that $G_{bx} = bG_x b^{-1}$ for any $b \in G$.
- ✓ 19. Let G be a group. Prove that $G/Z(G) \cong \text{Inn}(G)$, where $\text{Inn}(G)$ denotes the group of all inner automorphisms of G .
20. Let G be a group of order 273. Show that G has a cyclic subgroup of order 91.
- ✓ 21. In S_4 , find the centralizer of the permutation (123). Hence determine the size of its conjugacy class.

Group - C

Answer any *two* questions : $10 \times 2 = 20$

22. (a) Let G be a cyclic group. Prove that any subgroup of G is a characteristic subgroup.
- (b) Let G be a group such that $|G|=15$. Find the class equation of G .
- ✓ (c) Let G be a non-commutative group of order 27. Find $|Z(G)|$.
- ✓ (d) State the fundamental theorem for finite Abelian groups.

P.T.O.



23. (a) State Cauchy's theorem. Use Cauchy's theorem to prove that if a finite group G is a p - group, then $|G| = p^n$ for some positive integer n , where p is a prime.
- (b) Using the class equation, prove that the center of a finite p -group is non-trivial. 5+5
24. (a) If G be a group of order p^r , where p is a prime and $r \in \mathbb{Z}^+$, then prove that G has a non-trivial centre.
- (b) State the fundamental theorem of finite abelian groups. Use the theorem to classify all abelian groups of order 720 up to isomorphism. 5+(1+4)
25. (a) Let H be a subgroup of index n in a group G . By considering the action of G on the set of left cosets of H , prove that $G/Co_G(H)$ is isomorphic to a subgroup of S_n , where $Co_G(H) = \bigcap_{g \in G} gHg^{-1}$.
- (b) If $[G : H] = p$, where p is the smallest prime dividing $|G|$, prove that H is normal in G . 5+5

