2026

6th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 12-T

(Single Core Major)

(Group Theory-II)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *ten* questions : 2×10=20

1. "If G_1 and G_2 are two finite groups such that $|G_1| > |G_2|$, then $|Aut(G_1)| > |Aut(G_2)|$." — Check whether this statement is true or false. Give reaons.

2. Find all the characteristic subgroups of S_3.

3. Show that a cyclic group of order 4 cannot be expressed as an internal direct product of two of its subgroups of order 2.

4. Find the order of the element $(\bar{5},\bar{5})$ in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{16}$.

P.T.O.

5. Let G be a finite group that has only two conjugacy classes. Then show that $|G| = 2$.

6. Using Sylow's theorem prove that any group of order 63 is not simple.

7. Is $\mathbb{Z} \oplus \mathbb{Z}$ a cyclic group? Justify your answer.

8. Find the number of elements of order 10 in the group $\mathbb{Z}_{50} \oplus \mathbb{Z}_{25}$.

9. Show that the kernel of the group action is a subgroup.

10. Give an example of an infinite p-group, p being a prime.

11. Determine $Aut(Z)$, where Z denotes the infinite cyclic group under addition.

12. Let G_1, G_2 be commutative groups. Show that direct product $G_1 \times G_2$ is a commutative group.

13. Show that any group of order p^2 is commutative, where p is prime.

14. State Cauchy's theorem.

15. Find all Syllow 2-subgroups of A_4.

Group - B

Answer any ***four*** questions : 5×4=20

16. Let H and K be normal subgroups of a group G such that G=HK and $H \cap K = \{e\}$. Prove that $G \cong H \times K$.

17. Prove that the automorphism group of $(\mathbb{Z}_n, +)$ is isomorphic with U_n.

18. Let G be a group acting on a non-empty set S and $x \in S$. Define the stabilizer G_x of x. Then prove that $G_{bx} = bG_x b^{-1}$ for any $b \in G$.

19. Let G be a group. Prove that $G/Z(G) \simeq Inn(G)$, where $Inn(G)$ denotes the group of all inner automorphisms of G.

20. Let G be a group of order 273. Show that G has a cyclic subgroup of order 91.

21. In S_4, find the centralizer of the permutation (123). Hence determine the size of its conjugacy class.

Group - C

Answer any ***two*** questions : 10×2=20

22. (a) Let G be a cyclic group. Prove that any subgroup of G is a characteristic subgroup.

(b) Let G be a group such that $|G| = 15$. Find the class equation of G.

(c) Let G be a non-commutative group of order 27. Find $|Z(G)|$.

(d) State the fundamental theorem for finite Abelian groups.

3+3+2+2

P.T.O.

23. (a) State Cauchy's theorem. Use Cauchy's theorem to prove that if a finite group G is a p - group, then $|G| = p^n$ for some positive integer n, where p is a prime.

(b) Using the class equation, prove that the center of a finite p-group is non-trival. 5+5

24. (a) If G be a group of order p^r, where p is a prime and $r \in \mathbb{Z}^+$, then prove that G has a non-trivial centre.

(b) State the fundamental theorem of finite abelian groups. Use the theorem to classify all abelian groups of order 720 up to isomorphism. 5+(1+4)

25. (a) Let H be a subgroup of index n in a group G. By considering the action of G on the set of left cosets of H, prove that $G / Co_G(H)$ is isomorphic to a subgroup of S_n, where $Co_G(H) = \bigcap_{g \in G} gHg^{-1}$.

(b) If $[G : H] = p$, where p is the smallest prime dividing $|G|$, prove that H is normal in G. 5+5

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