B.Sc./6th Sem/MTM/26(NEP)

2026

6th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 11-T

(Single Core Major)

(Partial Differential Equations and Applications)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *ten* questions : 2×10=20

1. Define a quasi-linear first-order partial differential equation and give one example.

2. Find the partial differential equation of all spheres of constant radius a having their centres on the x-axis.

3. Find the characteristic curves of the PDE, $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$.

P.T.O.

4. Let $z(x,y)$ be the solution of $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z$ satisfying the condition $z(x,y)=1$ on the circle $x^2+y^2=1$. Then find the value of $z(2, 2)$.

5. Determine the region in which the PDE, $u_{xx}+x^2u_{yy}=0$ is hyperbolic, parabolic or elliptic.

6. Classify the second-order linear PDE,
$$u_{xx}+2u_{xy}+u_{yy}=0$$ and state is type.

7. Give the geometrical interpretation of the solution of a first-order partial differential equation.

8. State the Cauchy-Kowalevskaya theorem.

9. Write Kepler's second law of planetary motion.

10. Write the differential equation of the central orbit in pedal form.

11. Find the general solution of $p+q=0$.

12. Show that for a particle moving under a central force, the angular momentum per unit mass h is given by
$$h=pv$$
where p is the perpendicular distance from the center of force to the tangent of the orbit and v is the velocity of the particle.

13. Solve : $(x+y)p+(x-y)q=0$.

14. Find the auxiliary equations of : $(x^2)p+(y^2)q=xy$.

15. Form the PDE by eliminating the arbitrary function :
$$u=f\left(\frac{y}{x}\right).$$

Group - B

Answer any ***four*** questions : 5×4=20

16. Using the method of separation of variables, solve
$$\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=3z$$
where $z(0,y)=e^{-y}+e^{-4y}$.

17. Find the integral surface of the linear PDE
$$x(y^2+z)\frac{\partial z}{\partial x}+y(x^2+z)\frac{\partial z}{\partial y}=(x^2+y^2)z$$
which contains the straight line $x-y=0,\ z=1$.

18. Find the characteristics of the PDE
$$\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}+\frac{\partial z}{\partial x}-2\frac{\partial z}{\partial y}=0.$$

19. For a particle moving in a central orbit under the inverse-square law $\frac{\mu}{r^2}$, prove that the velocity v at any distance r is given by
$$v^2=\mu\left(\frac{2}{r}-\frac{1}{a}\right).$$

P.T.O.

20. A particle moves in a plane under a central force directed towards a fixed point. Show that if $u = \frac{1}{r}$, then the differential equation of the orbit is $\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}$, where h is the angular momentum per unit mass.

21. Show that the motion of a planet under gravitational attraction is a central force motion. Explain its implications.

Group - C

Answer any ***two*** questions : 10×2=20

22. (a) Reduce the equation $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ to its canonical form and hence solve it.

    (b) Derive the one-dimensional wave equation. 6+4

23. (a) Solve the heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, 0 < x < L$$

with boundary conditions

$$u(0,t) = 0,\ u(L,t) = T_0$$

and initial condition $u(x,0) = 0$.

(b) Solve the Cauchy problem

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty$$

with initial conditions

$$u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = g(x).$$

5+5

24. Solve the following initial-boundary value problem for the wave equation by the method of separation of variables :

$$u_{tt} = c^2u_{xx},\quad 0 < x < l, t > 0$$
$$u(x,0) = 0,\quad u_t(x,0) = g(x),\quad 0 \le x \le l$$
$$u(0,t) = 0,\quad u(l,t) = 0,\quad t \ge 0.$$

Hence find the solution when $g(x) = x(l - x), 0 \le x \le l$.

25. (a) A particle describes an ellipse under a force which is always directed towards the centre of the ellipse. Find the law of force.

    (b) A machine gun of mass M_0 stands on a horizontal plane and contains a shot of total mass m which is fired horizontally at a uniform rate with constant velocity u relative to the gun. Discuss the motion. 6+4

———