2026

6th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MI 6-T

(Single Core Minor)

(Numerical Methods)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : 2×10=20

1. Write the main drawbacks of Newton-Raphson method. How can it be removed?

2. Can you derive Trapezoidal rule of numerical integration from general quadrature formula based on Newton's forward difference interpolation formula? If so, derive this.

3. Derive an iteration function to solve the equation $\cos x = xe^x$.

P.T.O.

4. Prove or disprove that $\Delta\left\{\frac{1}{f(x)}\right\}=\frac{1}{\Delta f(x)}$, where Δ is forward difference operator.

5. Write the main drawbacks of Euler's method. Is this method a single step explicit method?

6. Find the relative error in the computation of $x-y$ for $x = 12.05$ and $y = 8.02$ having absolute error $\Delta x = 0.005$ and $\Delta y = 0.001$.

7. Why Regula-Falsi method is called the method of false position?

8. If $u=\frac{8xy^2}{z^3}$ and errors in x, y, z be 0.001; compute the maximum absolute and relative errors when $x=y=z=1$.

9. If $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$ are errors in computing $x_1, x_2, \ldots, x_n$ respectively, then show that the relative error in $x_1^m x_2^m \ldots x_n^m$ does not exceed

$$m\left(\left|\frac{\Delta x_1}{x_1}\right|+\left|\frac{\Delta x_2}{x_2}\right|+\ldots+\left|\frac{\Delta x_n}{x_n}\right|\right).$$

10. Prove that $\nabla E^{\frac{1}{2}} = E^{\frac{1}{2}}\nabla = \delta$.

11. Write down the equation $x^3+2x=10$ in the form of $x=\varphi(x)$, such that the iterative scheme about $x=2$ converges.

12. If $f(x)$ is a quadratic polynomial, deduce that

$$\int_1^3 f(x)dx = \frac{1}{12}[f(0)+22f(2)+f(4)].$$

13. Define the divided difference and show that the order of the arguments in a divided difference is immaterial.

14. What is partial pivoting? How many additional numbers of operation are required to implement it while solving a system of equation using Gauss elemination method?

15. Does the following system of equations satisfy the sufficient condition of convergence of Gauss-Seidel iteration?

$$x-10y+z=12;\quad 2x+7y+20z=10;\ 10x-y-z=11.$$

Group - B

Answer any ***four*** questions : 5×4=20

16. State Newton-Gregory forward difference interpolation formula. Using this formula, compute $f(1.02)$ from the following table :

x	1.00	1.10	1.20	1.30
$f(x)$	0.8415	0.8912	0.9320	0.9636

Is Gregory's backward difference interpolation formula useful for this computation? 1+3+1

P.T.O.

17. Find the largest (in magnitude) eigenvalue (correct up to 3 decimal places) and corresponding eigenvector of the matrix bu using power method

$$\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

18. Compute $y(0.2)$, from the equation $\frac{dy}{dx} = x - y$, $y(0) = 1$, taking $h = 0.1$, by Runge-Kutta method, correct to five decimal places. RK2

19. Apply method of bisection to find a real root of $x^3 + x^2 - 1 = 0$, correct to two significant figures.

20. Define the Lagrangian fuctions in connection to Lagrange's polynomial inerpolation. Show that these are invariant under a linear tranformation. If $x_0, x_1, ..., x_n$ are the interpolating points and $l_i(x), i = 0,1,2,\cdots,n$ are the Lagrangian functions, prove that $\sum_{i-0}^{n} l_i(x) = 1$. 1+2+2

21. Describe the method of false positions for finding a real root of an equation $f(x) = 0$ and obtain the corresponding iteration formula. Discuss its disadvantages in comparison to Newton-Raphson method. 4+1

Group - C

Answer any *two* questions : 10×2=20

22. (a) Solve the system

$$9x + 2y + 4z = 20,$$
$$x + 10y + 4z = 6,$$
$$2x - 4y + 10z = -15$$

by Gauss-Seidal's method.

(b) Solve the following system of linear equations by Gauss-elimination method :

$$x - y - z = 1,$$
$$2x - 3y + z = 1,$$
$$3x - y - z = 2.$$

5+5

23. (a) Establish the Newton's backward interpolation formula for $(n+1)$ equispaced interpolating points and the associated remainder term. When is this formula used?

(b) A function f defined on [0, 1] is such that $f(0) = 0$, $f\left(\frac{1}{2}\right) = -1$, $f(1) = 0$. Find the quadratic polynomial $p(x)$ which agrees with $f(x)$ for $x = 0, \frac{1}{2}, 1$. Further, if $f'(x) \le 1, 0 \le x \le 1$, show that $|f(x) - p(x)| \le \frac{1}{12}$ for $0 \le x \le 1$.

(4+1)+(2+3)

P.T.O.

24. What do you understand by open and closed type numerical integration formula? By integrating Newton's forward interpolation formula, obtain the basic form of Simpson's one-third rule for numerical integration. Also obtain the composite form of this rule. Write down the error formula in each case. Show that the degree of precision of Simpson's one-third rule for both simple and compound case is 3. 1+3+2+2+2

25. (a) Solve the following system of equations by LU decomposition method :

$$2x-3y+10z=3,$$
$$-x+4y+2z=20,$$
$$5x+2y+z=-12.$$

(b) Prove for equally spaced interpolating points $x_i = x_0 + ih \ (h>0,\ i=1,2,\dots)$;

$$\Delta^k y_0 = \sum_{i=0}^{k}(-1)^i \binom{k}{i} y_{k-i}.$$ 6+4

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B.Sc./6th Sem/MTM/26(NEP)

2026

6th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MI 6-T

(Multidisciplinary Studies Minor)

(Probability and Statistics)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : $2\times10=10$

1. Show that the probability that two numbers, chosen at random, will be prime to each other is $\frac{6}{\pi^2}$.

2. In a game of bridge, the entire deck of 52 cards is dealt out to 4 people. Find the probability that one of the players receives all 13 hearts.

3. Suppose that the probability of a new born baby, a girl is 0.55. In a family of 8 children, calculate the probability that the number of girls lies in the interval [3, 7].

P.T.O.

4. If X be a continuous random variable then prove that $P(X = a) = 0$ for any real number a.

5. Prove that for any distribution, the first absolute moment about the mean cannot exceed the standard deviation.

6. Find the conditional probability that two threes occur in throwing two unbiased dice, given that the total is divisible by 3.

7. Find the moment generating function of a random variable X uniformly distributed over $(-1,1)$.

8. Distinguish between regression lines and regression curves for a bivariate distribution.

9. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

10. If the correlation coefficient 'ρ' of two random variables X and Y is zero, then are X, Y independent? Justify your answer.

11. State the central limit theorem for equal components. Does it imply the law of large numbers for equal components?

12. Find the sampling distribution of the sample mean for the normal population.

13. Define the simple hypothesis and state Neuman-Pearson lemma in testing of it against a simple alternative.

14. What is standard error? Obtain the standard error of mean when population is large.

15. Define an unbiased and consistent estimate of a parameter in a population distribution.

Group - B

Answer any ***four*** questions : 5×4=20

16. Two points are at random on a given straight line of length a. Prove that the probability of their distance exceeding a given length $c(< a)$ is $\left(1-\dfrac{c}{a}\right)^2$.

17. Prove that the maximum likelihood estimates of the parameter α of the population having density function $f(x)=\dfrac{2(\alpha-x)}{3\alpha^2}$, $0<x<\alpha$ for a sample x_1 of unit size is $2x_1$. Test whether the estimate is biased or not.

18. Two points are chosen at random in the interval (0,1). Find the probability that the three parts into which the interval is divided, can form a triangle.

19. X and Y are independent random variables each uniformly distributed in (0,1). Find the probability density functions of $X+Y$ and $|X-Y|$.

20. If X_n is the number of successes in n trials and $p=\dfrac{1}{2}$ is the probability of a success in one trial, then compute how many independent Bernoulli trials are required to ensure with the probability of $\left|\dfrac{1}{n}X_n-p\right|\geq 0.1$ is less than 0.025.

P.T.O.

21. If $X_1, X_2, \cdots, X_n$ is a random sample obtained from the density function $f(x,\theta) = \begin{cases} 1, \text{ if } \theta < x < \theta+1 \\ 0, \quad \text{elsewhere} \end{cases}$

then show that the sample mean $\bar{X}$ is an unbiased and consistent estimator of $\theta + \frac{1}{2}$.

Group - C

Answer any ***two*** questions : 10×2=20

22. (a) The joint probability density function of a two-dimensional random variables (X, Y) is given by
$f(x,y) = e^{-(x+y)}, 0 \le x, y < \infty$. Find
$P(X < Y \mid X < 2Y)$ and $P(1 < X + Y < 2)$.

(b) State limit theorem for characteristic functions. Use it to obtain Poisson distribution as a limiting case of bionomial distribution. (3+2)+(1+4)

23. (a) Nine patients to whom a certain drug is administered registered the following rise in blood pressure in mm of Hg: 3,7,4,–1,–3,6,–4,1,5. Test the hypothesis that the drug does not raise blood pressure at 10% significance level assuming that the sample is chosen from a normal population. Given that $P(t > 1.86) = 0.05$ for degrees of freedom.

(b) Suppose (X, Y) is a two-dimensional random variable. Show that $\{E(X,Y)\}^2 \le E(X^2)E(Y^2)$.

Deduce that $-1 \le \rho(X,Y) \le 1$ where ρ is the correlation coefficient between X and Y. 5+(2+3)

24. (a) If X is a normal (m,σ) variate, find the density function of $\frac{X-m}{\sigma}$.

(b) A fair coin is tossed three times. The random variables X, Y and Z denote the number of heads, number of occurrences of two consecutive heads and the number of heads preceding the first tail, respectively. Find the probability distributions of X, Y, Z.

(c) For a random sample $\{X_1, X_2, ..., X_n\}$ drawn from a $N(m,\sigma)$ population. Show that the sample mean $\bar{X}$ follows $N(m, \sigma/\sqrt{n})$ distribution.

2+4+4

25. (a) Show that the probability ꞁhat the number of tails in 20 times throws with a fair coin lies between 9 and 11 is $2F(2\sqrt{5})-1$, where $F(x)$ is the standard normal distribution function. Compare the result with the lower limit given by Tchebycheff inequality. Given that $F(2\sqrt{5}) = 0.99996$.

(b) Find the median and mode of normal (m,σ) distribution. 5+5

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