 B.Sc./6th Sem/MTM/26(NEP)

2026

6th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MDSE 2-T

(Single Core Major Elective-2)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

(Probability and Statistics)

Group - A

Answer any *ten* questions : 2×10=20

1. Define the sample space S of a random experiment and give one example.

2. If $P(A)=0.4, P(B)=0.5, P(A \cap B)=0.2$, find $P(A \cup B)$.

3. State the addition theorem for two events A and B.

4. For a discrete random variable X with probability mass function $p_x(x)$, write the formula for $E(X)$.

5. Write the moment generating function of a random variable X.

P.T.O.

6. If $X \sim Bin(n, p)$, write $E(X)$ and $Var(X)$.

7. Write the probability mass function of $X \sim Poisson(\lambda)$.

8. If $X \sim Exp(\lambda)$, write its distribution function $F_X(x)$.

9. Define the marginal density $f_X(x)$ in terms of the joint density $f_{XY}(x, y)$.

10. State the condition for independence of two continuous random variables X and Y.

11. Define the correlation coefficient ρ_{xy}.

12. State Chebyshev's inequality for a random variable with mean μ and variance σ^2.

13. State the central limit theorem for independent and identically distributed random variables with finite variance.

14. Define the *n*-step transition probability $p_{ij}^{(n)}$ of a Markov chain.

15. What is an unbiased estimator of a parameter θ?

Group - B

Answer any ***four*** questions : 5×4=20

16. Let $X \sim Geometric(p)$ with $P(X = k) = p(1-p)^{k-1}$, $k = 1, 2, \ldots$ Find $E(X)$ and $Var(X)$.

17. A continuous random variable X has probability density function $f(x) = cx(1-x), 0 < x < 1$ and $f(x) = 0$ otherwise. Find c and $E(X)$.

18. The joint density of (X, Y) is $f(x, y) = 2, 0 < x < y < 1$ and 0 otherwise. Find the marginal densities and $E(X \mid Y = y)$.

19. If the joint moment generating function of (X, Y) is $M_{XY}(s, t) = \exp(2s + 3t + 2s^2 + (9/2)t^2 + 3st)$, find $E(X)$, $E(Y)$, $Var(X)$, $Var(Y)$ and $Cov(X, Y)$.

20. If $E(X) = 10$ and $Var(X) = 4$, use Chebyshev's inequality to find an upper bound for $P(|X - 10| \geq 6)$.

21. For a Markov chain with transition matrix $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$, compute P^2 and $P_{12}^{(2)}$.

Group - C

Answer any ***two*** questions : 10×2=20

22. For $X \sim Bin(n, p)$, derive the moment generating function $M_X(t) = (1 - p + pe^t)^n$ and hence find $E(X)$ and $Var(X)$. Also discuss the limiting Poisson form when $n \to \infty, p \to 0, np \to \lambda$.

P.T.O.

23. Let the joint probability density function of (X, Y) be $f(x,y) = x + y,\ 0 < x < 1,\ 0 < y < 1$ and 0 otherwise. Find the marginal densities, conditional density of X given $Y = y$, $E(X \mid Y = y)$, $Cov(X,Y)$, and decide whether X and Y are independent.

24. State and prove Chebyshev's inequality. Use it to prove the weak law of large numbers for a sequence of independent and identically distributed random variables having finite variance.

25. A random sample of size $n = 25$ from a normal population with known standard deviation $\sigma = 5$ gives sample mean $\bar{x} = 52.4$. Test $H_0 : \mu = 50$ against $H_1 : \mu \neq 50$ at the 5% level of significance. Also construct a 95% confidence interval for μ.

OR

(Logic and Graph Theory)

Group - A

Answer any ***ten*** questions : $2\times10=20$

1. Write the inclusion-exclusion principle for two finite sets.

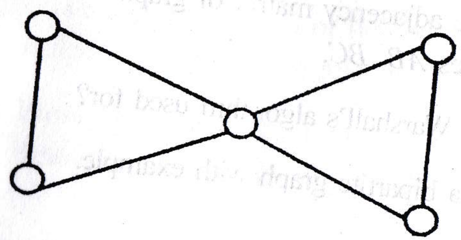
2. Explain the predicates and quantifiers in discrete mathematics.

3. Define partial order relation. Give an example.

4. If $D_1 = \{2,3,4\}, D_2 = \{6,7,8\}$ and $D_3 = \{3,4,5\}$ are the domains of the variables x, y and z respectively, then find the truth value of the following quantified predicate. $\exists x \forall y \exists z\, p(x,y,z)$, where $p(x,y,z) : x^2 + y^2 < 3z^2$.

5. Define Eulerian circuit. Draw a graph which is Hamiltonian but not Eulerian.

6. For the following finite and connected planar graph, verify the Euler's formula $v - e + f = 2$, where v is the number of vertices, e is the number of edges and f is the number of faces (regions bounded by edges, including the outer, infinitely large region).

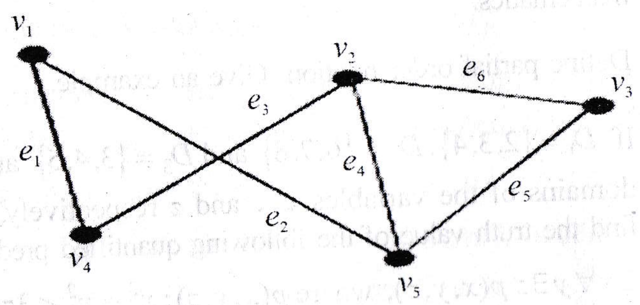


P.T.O.

7. What is the difference between multi-graph and pseudo-graph? Give examples.

8. For a graph G with n vertices and m edges, show that $\sum_{i=1}^{n} d(v_i) = 2m$, where $d(v_i)$ is the degree of the vertex v_i.

9. Find the incidence matrix of the following graph.



10. Simplify : $\neg(p \vee q) \wedge (p \vee \neg q)$

11. Determine the truth value of :

$$(p \to q) \wedge (q \to p) \text{ when } p = T, q = F.$$

12. Determine whether a graph with 5 vertices and 4 edges can be a tree.

13. Write the adjacency matrix of graph with vertices A, B, C and edges AB, BC.

14. What is Warshall's algorithm used for?

15. Define a bipartite graph with example.

Group - B

Answer any ***four*** questions : 5×4=20

16. (a) Prove that $p \to q, q \to r \models p \to r$.

(b) What is tautology? Show that $(p \wedge q) \to (p \vee q)$. 2+3

17. (a) Define an n-ary relation. Give example of a unary, binary and ternary relations.

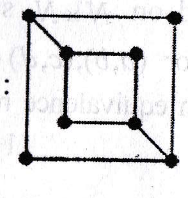
(b) If R is an equivalence relation in a set A, then show that R^{-1} is also an equivalence relation. 2+3

18. If ρ be an equivalence relation on a set S and $a, b \in S$. Then prove that $cl(a) = cl(b)$ iff $a \rho b$.

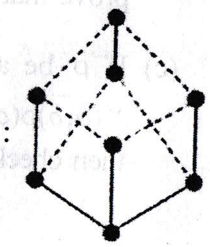
19. Define a rooted tree. Prove that a tree with n vertices has $n - 1$ edges. 1+4

20. Define graphs isomorphism. Check whether the following two graphs are isomorphic or not. 2+3

G_1 :



G_2 :



21. (a) Prove that intersection of two equivalence relations is again an equivalence relation.

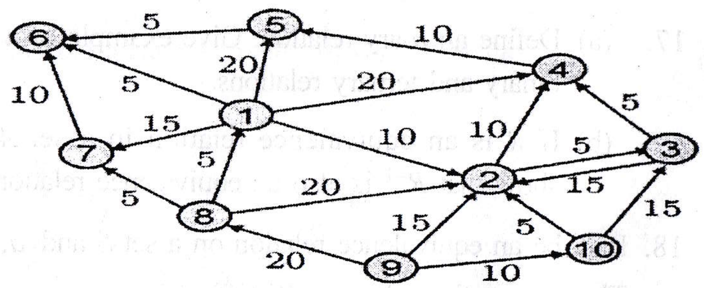
(b) Prove that every equivalence relation on a set determines a partition of the set. 2+3

P.T.O.

Group - C

Answer any *two* questions : 10×2=20

22. Define weighted shortest path between two vertices. Apply Dijkstra's algorithm to the graph given below and find the shortest path from the vertex 9 to the vertex 5.



2+8

23. (a) Define Cartesian product of n non-empty sets. If $O(A)=n$, and $O(B)=m$, then find $O(P(A\times B))$.

(b) Let A, B be subsets of a universal set S. Then prove that $A = B$ iff $A \Delta B = \phi$.

(c) If ρ be a relation defined on $N\times N$ such that "$(a,b)\rho(c,d)$ iff $ad=bc$" for $(a,b),(c,d)\in N\times N$, then check whether ρ is an equivalence relation? (1+1)+3+5

24. (a) What is universal quantifier? Symbolize the following sentences using predicates and quantifiers :

(i) Everybody is not rich.

(ii) Every integer is divisible by 6 iff it is divisible by both 2 and 3.

(b) Test the logical validity of the following arguments :
All men are mortal. Socrates is a man. Therefore, Socrates is a mortal.

(c) Write down the De Morgan's laws and absorption laws in the set of statement formulas. After then prove any one of the De Morgan's laws and the absorption laws by truth table. (1+2)+3+(1+3)

25. (a) Simplify using laws of logic :

$[(p\vee q)\wedge(\neg p\vee r)]\wedge(p\vee\neg r)$

(b) Prove using logical equivalences :

$(p\to q)\wedge(p\to r)\equiv p\to(q\wedge r)$ 5+5

P.T.O.

OR

(Portfolio Optimization)

Group - A

Answer any ***ten*** questions : 2×10=20

1. Define financial market.
2. What is meant by investment objective?
3. Distinguish between risk and return.
4. Define systematic risk.
5. What is unsystematic risk?
6. What is a risk-free asset?
7. Define mutual fund.
8. What is diversification?
9. Define portfolio.
10. Write the formula for expected portfolio return.
11. What is meant by efficient frontier?
12. Define short selling.
13. State the two-fund theorem.
14. What is Capital Market Line (CML)?
15. Define beta of a security.

Group - B

Answer any ***four*** questions : 5×4=20

16. An investor invests 40% in Asset A with expected return 10% and 60% in Asset B with expected return 15%. Calculate the expected return of the portfolio.
17. Discuss different types of risks in investment.
18. Explain diversification and its importance in portfolio management.
19. Describe the Markowitz mean-variance model.
20. Explain the efficient frontier with a neat diagram.
21. Distinguish between Capital Market Line and Security Market Line.

Group - C

Answer any ***two*** questions : 10×2=20

22. An investor wants to construct a portfolio using two securities A and B.

Security	**Expected Return**	**Standard Deviation**	**Weight**
A	12%	10%	40%
B	18%	20%	60%

Correlation coefficient between A and $B = 0.5$

Calculate :

(a) Expected return of the portfolio

(b) Portfolio risk (standard deviation)

P.T.O.

23. Discuss Capital Asset Pricing Model (CAPM) and derive the Security Market Line.

24. Explain Capital Market Theory and discuss the role of risk-free assets in portfolio construction.

25. The following information is available for a security :

- Risk-free rate = 5%
- Expected market return = 13%
- Beta of the security = 1.4

Calculate :

(a) Expected return using CAMP

(b) Whether the security is overvalued or undervalued if the actual return is 18%.