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PG (CBCS)
M.SC. Semester- IV Examination, 2026
APPLIED MATHEMATICS
PAPER: MTM 405A
(OPERATION RESEARCH MODELLING II)

Full Marks: 20 **Time: 1 Hour**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **TWO** of the following questions: 2×2=4

a) What do you mean by memoryless channel and channel matrix?
b) Prove that the entropy function $H(p_1, p_2 \dots \dots \dots p_n)$ is continuous in $p_k, \forall\ 0 \leq p_k \leq 1$
c) Write down the properties of reliability function.
d) Show that energy function H is continuous.

GROUP-B

2. Answer any **TWO** of the following questions: 2×4=8

a) The two finite probability schemes are given by $(p_1, p_2, p_3, \dots, p_n)$ and $(q_1, q_2, q_3, \dots, q_n)$ with $\sum_{i=1}^{n} p_i = \sum_{i}^{n} q_i$, then show that $\sum_{i=1}^{n} p_i \log p_i \leq -\sum_{i}^{n} p_i \log q_i$, with inequality holds if and only if $p_i = q_i$ for all i.

b) Consider a binary channel with input symbols $A = \{0,1\}$, output symbol $B = \{0,1\}$ and the channel matrix $\begin{pmatrix} 2/3 & 1/3 \\ 1/10 & 9/10 \end{pmatrix}$. Find the value of all joint probabilities.

c) The system connected in series consists of three independent parts A, B, and C, which have MTBF of 100, 400, and 800 hours, respectively. Find MTBF of the system and reliability of the system for 30 hours. How much of MTBF of the part A has to be increased to get an improvement of the MTBF of the system by 30%?

d) Find the stationary path $x = x(t)$ for the functional

$$J = \int_0^1 (1 + \ddot{x}^2)\, dt$$

subject to the boundary conditions $x(0) = 0, \dot{x}(0) = 1, \dot{x}(1) = 1$.

(P.T.O)

GROUP-C

3. Answer any **ONE** of the following questions: 1×8=8

a) Let a car be driven from a stationary position on a horizontal way to a stationary position in a garage moving a total distance a. The available control for the driver is the accelerator and the break. Find the minimum time to bring the car to the stationary position at a distance α and the optimal control to the applied to the car.

b) Determine the different entropies for the channel. Also determine H(X), H(Y), H(X, Y) and H(Y/X) where the joint probability for the communication is given below

$P(x_i / y_i)$	Y_1	Y_2	Y_3	Y_4	Y_5
X_1	0.25	0	0	0	0.25
X_2	0.10	0.30	0	0	0.40
X_3	0	0.05	0.10	0	0.15
X_4	0	0	0.05	0.10	0.15
X_5	0	0	0.05	0	0.15
$P(y_j)$	0.35	0.35	0.20	0.10	0.05

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