B.Sc./5th Sem/MTM/25(NEP)

2025

5th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 9-T (Single Core Major)

[Multivariate Calculus]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : 2×10=20

1. Define a limit point of a set S in R^2.

2. Define continuity of a function $f(x,y)$ in a region S of its domain of definition $D \subset R^2$.

3. Define differentiability of a function $f(x,y)$ at a point (a, b) in its domain of definition $D \subset R^2$.

4. Check the existence of the limit, $\lim\limits_{(x,y)\to(0,0)} xy\dfrac{(x^2-y^2)}{(x^2+y^2)}$.

P.T.O.

5. Show that the function $f(x,y)=y^2+x^2y+x^4$ has a minima at (0, 0).

6. Define divergence and curl of a vector function.

7. What do you mean by conservative vector field?

8. Find the directional derivative of $\phi = xy^2z+4x^2z$ at (–1, 1, 2) in the direction $2\vec{i}+\vec{j}-2\vec{k}$.

9. A particle moves along a curve whose parametric equations are $x=e^{-t}$, $y=2\cos 3t$, $z=3\sin 3t$, where t is the time. Find the velocity and acceleration of the particle at any timne t.

10. What do you mean by bounded set in R^n?

11. Explain the area of a polar curve $\frac{1}{2}\int_{\theta=\alpha}^{\theta=\beta} r^2 \frac{d\theta}{dt} dt$.

12. Distinguish between the double limit and repeated limit to a function of double variable.

13. Find $\iint_R y\,dx\,dy$ over the part of the plane bounded by the lines $y = x$ and $y = 4x - x^2$.

14. Evaluate $\vec{\nabla}(\vec{\nabla}.\vec{A})$, when $\vec{A}=\frac{\vec{r}}{r}$, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. Where $r=|\vec{r}|$.

15. Change the order of integration $\int_0^1 dx\int_x^{\sqrt{x}} f(x,y)dy$.

Group - B

Answer any ***four*** questions : 5×4=20

16. State and prove a sufficient condition that a function $f(x, y)$ be continuous at a point $(a,b)\in R^2$.

17. Check the differentiability of the function
$$f(x,y)=\sqrt{|xy|} \text{ at } (0,0).$$

18. For a function $f(x, y)$ where $x = u^2v$ and $y = uv^2$, prove that
$$2x^2\frac{\partial^2 f}{\partial x^2}+2y^2\frac{\partial^2 f}{\partial y^2}+5xy\frac{\partial^2 f}{\partial x\partial y}=uv\frac{\partial^2 f}{\partial u\partial v}-\frac{2}{3}\left(u\frac{\partial f}{\partial u}+v\frac{\partial f}{\partial v}\right).$$

19. Verify Stoke's theorem for the vector function $\vec{V}=\left(x^2-y^2\right)\vec{i}+2xy\vec{j}$ round the rectangle bounded by the straight lines $x=0$, $x=a$, $y=0$, $y=b$.

20. Show that for a traingular region T bounded by $x=0$, $y=0$, $x+y=1$,
$$\iint_T x^{\frac{1}{2}}y^{\frac{1}{3}}(1-x-y)^{\frac{2}{3}}=\frac{\Gamma\left(\frac{5}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{9}{2}\right)}$$

P.T.O.

21. For a vector function $\vec{F}$, prove that

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)=\vec{\nabla}\left(\vec{\nabla}.\vec{F}\right)-\vec{\nabla}^2\vec{F}.$$

Group - C

Answer any ***two*** questions : 10×2=20

22. State and prove a sufficient condition for the equality of $\dfrac{\partial^2 f(x,y)}{\partial x\partial y}$ and $\dfrac{\partial^2 f(x,y)}{\partial y\partial x}$ at a point $(a,b)\in R^2$. Is the stated condition necessary? Justify. 1+5+4

23. (a) State and prove a sufficient condition that a function $f(x,y)$ be differentiable at a point $(a,b)\in R^2$.

(b) Show that the function

$f(x,y,z)=(x+y+z)^2-3(x+y+z)-24xyz+25$ has a minima at (1, 1, 1) and a maxima at (–1,–1,–1). 5+5

24. (a) For an ellipsoid $E:\ \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$, prove that

$$\iiint_E \sqrt{a^2b^2c^2-b^2c^2x^2-c^2a^2y^2-a^2b^2z^2}\,dxdydz=\frac{1}{4}\pi^2a^2b^2c^2.$$

(b) Show that for the function $f(x,y)$ the limit exists at (0, 0) but both repeated limits do not exist where

$$f(x,y)=\begin{cases} x\sin\left(\dfrac{1}{y}\right)+y\sin\left(\dfrac{1}{x}\right) & ,\ \text{for } xy\neq 0 \\ 0 & ,\ \text{for } xy=0 \end{cases}$$

6+4

25. (a) The plane $x+y+z=1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

(b) Evaluate $\iint_S\left(\vec{\nabla}\times\vec{A}\right)\cdot\hat{n}ds$, where $\vec{A}=\left(x^2+y-4\right)\hat{i}+3xy\hat{j}+\left(2xz+z^2\right)\hat{k}$, and S is the surface of the paraboloid $z=4-(x^2+y^2)$ above the xy plane. 6+4