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B.Sc./3rd Sem/MTM/25(NEP)

2025

3rd Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 3-T (Single Core Major)

[Real Analysis]

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

Group - A

Answer any *ten* questions : $2\times10=20$

1. Prove that there is no rational number whose square is 2.

2. Define interior of a set. Find the interior of the set $S=\{x\in R:1<x<3\}$.

3. Define "limit point" of a set. Give an example of an infinite set which does not have any limit point.

P.T.O.

4. Determine the glb and lub of the set $\left\{-2, \frac{3}{2}, -\frac{4}{3}, \frac{5}{4}, -\frac{6}{5}, \frac{7}{6}, -\frac{8}{7}, \frac{9}{8}, \ldots\right\}$, if they exist.

5. Find the derived set of the set $\left\{\frac{(-1)^m}{m} + \frac{\frac{1}{n}}{m}, m, n = 1, 2, 3, \ldots\ldots\right\}$.

6. Determine the upper limit μ and lower limit λ of the sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$.

7. Use root test to examine the convergence of the series $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\ldots\ldots$

8. Examine whether the series $\sum_{n=1}^{\infty}\frac{n}{n+1}$ is convergent or not.

9. Define null sequence. Give an example of null sequence.

10. Define countability of a set. Show that the set Q of rational numbers is countable.

11. Prove that union of an arbitrary collection of open sets is an open set.

12. Find the sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2n+1}{n(n+1)}$.

13. State Leibnitz's Test of convergence for an alternating series.

14. Define bounded sequence. Give an example of bounded sequence.

15. Find a subsequence of the sequence $\{x_n\}$ where $x_n = \frac{1}{n}$.

Group - B

Answer any *four* questions : 5×4=20

16. Prove that the intersection of finite number of open sets is an open set. Show by an example that intersection of an arbitrary collection of open sets may not be open.

17. Prove that

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\frac{1}{\sqrt{n^2+3}}+\cdots\cdots+\frac{1}{\sqrt{n^2+n}}\right)=1.$$

18. Use Cauchy's criterion to prove that $\{x_n\}$ does not converge, where $x_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}$.

P.T.O.

19. Discuss the convergence of the following series :

$$\left(\frac{1}{2}\right)^2+\left(\frac{1.3}{2.4}\right)^2 x+\left(\frac{1.3.5}{2.4.6}\right)^2 x^2+\dots\dots\dots\dots\dots\dots, x>0.$$

20. If a sequence $\{x_n\}$ of real numbers is monotone increasing and bounded above, then prove that it converges to its exact upper bound.

21. Let $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ be two series with non-negative terms and suppose that there exist an integer N such that $u_n \le v_n$, $\forall n > N$. Then prove that

(i) $\sum_{n=1}^{\infty} u_n$ converges if $\sum_{n=1}^{\infty} v_n$ converges, and

(ii) $\sum_{n=1}^{\infty} v_n$ diverges if $\sum_{n=1}^{\infty} u_n$ diverges.

Group - C

Answer any *two* questions : 10×2=20

22. (a) Define Cauchy sequence and give an example of it. State and prove Cauchy's general principle of convergence of a sequence of real numbers.

(b) Prove that the sequence $\{x_n\}$ defined by $x_1=\sqrt{2}$, $x_{n+1}=\sqrt{2x_n}$, for $n\ge 1$, converges to 2. 5+5

23. (a) Prove that if a series $\sum_{n=1}^{\infty} a_n$ of real constants is absolutely convergent then it converges.

(b) Prove that a monotone increasing sequence, if bounded above, is convergent and it converges to the least upper bound. 5+5

24. (a) What do you mean by conditional convergence of a series? Show that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots\dots\dots\dots$ is conditionally convergent.

(b) If x and y are two real numbers such that $x<y$, then show that there exists a rational number r where $x<r<y$. 5+5

25. (a) Prove that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\dots+\frac{1}{n^p}+\dots$ is convergent if $p>1$ and divergent if $p\le 1$.

(b) If $\sum_{n=1}^{\infty} u_n$ be a series of positive terms, and $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\rho$, then prove that $\sum_{n=1}^{\infty} u_n$ converges if $\rho<1$ and $\sum_{n=1}^{\infty} u_n$ diverges if $\rho>1$.

(c) Prove that the series $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots\dots$ is convergent and find its sum. 4+3+3