2025

1st Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 1-T (Single Core Major)

(Calculus, Geometry and Ordinary Differential Equation)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : 2×10=20

1. Find the center of a conic represented by a second degree equation.

2. Solve $p^3 - 4xyp + 8y^2 = 0$.

3. Find the envelop (if any) of the line $x\cos\alpha + y\sin\alpha = 4$.

4. Find the eccentricity and the vertex of the conic $r = 3\sec^2\frac{\theta}{2}$.

P.T.O.

$m^2 - m - 2 = 0$

$(m+1)(m-2)$

5. Sketch the graph of $y = -\log(-x)$.

6. $\lim_{x \to 0+} (\sin x)^{2\tan x}$.

7. Find the Asymptotes of the curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

8. Discuss the nature of the conic represented by

$$3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0.$$

9. Define general solution and singular solution of a differential equation.

10. Find the values of the constant λ such that

$$\left(2xe^y + 3y^2\right)\frac{dy}{dx} + \left(3x^2 + \lambda e^y\right) = 0$$ is exact.

11. Using Leibnitz theorem, find the value of the n^{th} derivative for $y = e^{m\sin^{-1}x}$ at $x = 0$.

12. Find the points of inflexion of the curve $y = (\log x)^3$.

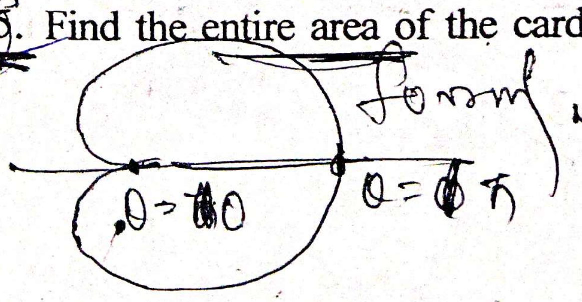
13. Show that the equation

$$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$$

represents a cone.

14. Find the general equation of a quadratic cone containing all three axes.

15. Find the entire area of the cardioid $r = a(1 - \cos\theta)$.



Group - B

Answer any *four* questions. 5×4=20

16. Prove that

$$J_{m,n} = \int_0^{\frac{\pi}{2}} \cos^m x \sin^n x dx = \frac{1}{2^{m+1}}\left[2+\frac{2^2}{2}+\frac{2^3}{3}+\frac{2^4}{4}+\ldots+\frac{2^m}{m}\right].$$

17. Prove that $e^{-x}D^n\left(x^n e^x\right) = \sum_{r=0}^{n}\left[\frac{n!}{(n-r)!}\right]^2 \frac{x^{n-r}}{r!}$.

18. Reduce the equation $4x^2+4xy+y^2+4x+y-15=0$ to its canonical form and determine the nature of the conic.

19. If the astroid $x^{2/3}+y^{2/3}=c^{2/3}$ is the envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$, prove that the parameters a and b are connected by the relation $a^2+b^2=c^2$.

20. Solve $\left(1-y^2+\frac{y^4}{x^2}\right)p^2-2\frac{y}{x}p+\frac{y^2}{x^2}=0$.

21. Determine the equation of the cubic which has the same asymptotes as the curve $x^3-6x^2y+11xy^2-6y^3+5x-3y+3=0$ and which passes through the points (0, 0), (1, 2), (2, 1).

P.T.O.

Group - C

Answer any *two* questions : 10×2=20

22. (a) Show that if y_1 and y_2 be solutions of equation $\frac{dy}{dx}+P(x)y=Q(x)$ and $y_2=zy_1$, then prove that

$$z=1+ae^{-\int\frac{Q(x)}{y_1}dx}, \quad a \text{ is an arbitrary constant.}$$

(b) Show that the points of inflection of the curve $y^2=(x-a)^2(x-b)$ lie on the line $3x+a=4b$.

5+5

23. (a) Prove that

$$(m+1)J_{m,n}=-\cos^m x\cos nx+mJ_{m-1,n-1} \quad \text{when}$$

$$J_{m,n}=\int\sin nx\cos^m x\,dx.$$

(b) A circle of a given diameter d passes through the focus of a given conic and cuts it in four points whose distances from the focus are r_1, r_2, r_3, r_4. Prove that

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}=\frac{2}{l} \text{ and } r_1r_2r_3r_4=\frac{d^2l^2}{e^2}. \quad 4+6$$

24. (a) If a_1, a_2, ..., a_n be all positive real numbers, find the value of

$$\lim_{x\to\infty}\left\{x-\sqrt[n]{(x-a_1)(x-a_2)\ldots(x-a_n)}\right\}.$$

(b) Find the ranges of values of x in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave or convex. Also find the points of inflexion. 5+5

25. (a) Prove that the equations

$$2x = ae^{2\phi}, 2y = be^{\phi}\cosh\theta, z = ce^{\phi}\sinh\theta$$

determine a hyperbolic paraboloid and that $\theta + \phi$ is constant for a generator of one system and $\theta - \phi$ is constant for a generator of the other system.

(b) Find the equation of the cylinder with guiding curve $2y^2 + 3z^2 = 1, x = 0$ and generators parallel to $x = y = z$. 7+3