2025

5th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MI 5-T (Single Core Minor)

[Numerical Method]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : 2×10=20

1. Find the approximate value of $\frac{1}{4}$ correct up to three significant figures and find the absolute error.

2. If $y = x^3 - 2x^2 - 1$; step size $4 = 1$, find Δy.

3. Evaluate $\left(\frac{\Delta}{E}\right)x^2$ taking $h = 1$.

4. Prove that $E[\Delta f(x)] = \Delta[Ef(x)]$.

5. Write the Weddle Rule to evaluate $\int_a^b f(x)dx$ tking six subintervals.

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6. Write the Gauss-Jacobi iterative formula for finding the solution of following system of equations :

$$x_1 + 3x_2 = 13;\ 2x_1 - x_2 = 7.$$

7. Using Euler Method, find the value $y(0, 1)$ from $\frac{dy}{dx} = \frac{1}{x+y}$ with $y(0) = 1$ and $h = 0.1$.

8. If in a root finding problem, the fixed point iteration is given by $x_{n+1} = \frac{x_n}{4} + \frac{4}{x_n}$ then write the corresponding root finding equation?

9. If $f(x) = x^3$, then find the second order divided difference for the points x_0, x_1, x_2.

10. Write the error term in approximation $f(x)$ by a interpolating polynomial, where the values of $f(x)$ are known at $(n + 1)$ distinct points $x_0, x_1, ..., x_n$.

11. Write the iterative formula to find the square root of N using Newton-Raphson method, N is a real number.

12. Write the advantages and disadvantages of Lagranges interpolation method.

13. Write the first approximation to the eigenvector corresponding to the dominant eigenvalue of the matrix $\begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{pmatrix}$, taking the initial approximation $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

14. Write the condition for convergence of fixed point iteration method.

15. What is rate of convergence of Newton-Raphson method? Is it faster than Regular-Falsi method?

Group - B

Answer any ***four*** questions : 5×4=20

16. (a) Prove that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$.

(b) If $f(1) = 2,\ f(2) = 4,\ f(3) = 8$, find $f(5)$. 2+3

17. Describe Regula-Falsi method to find a simple real root of $f(x) = 0$. Hence discuss the comparison between the Regula-Falsi Method and Secant Method.

18. Find a real root of the equation $x^3 + 3x - 5 = 0$ correct up to four significant figures, by Newton-Raphson method.

19. Solve the following system of equations by Gauss-Jacobi method (correct up to two decimal places) :

$$1.02x_1 - 0.05x_2 - 0.10x_3 = 0.80$$
$$-0.11x_1 - 0.12x_2 + 1.04x_3 = 1.34$$
$$-0.11x_1 + 1.03x_2 - 0.05x_3 = 0.85$$

20. Establish the Trapezoidal rule for computing the value of the integral $\int_{x_0}^{x_1} f(x)dx$ for a single interval. Hence,

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obtain composite Trapezoidal Rule by dividing the interval $[x_0, x_1]$ into n subintervals. 3+2

21. Find the value of $f(.201)$ by Newton's Forward Interpolation formula from the followng data (correct up to three decimal places) :

x	.200	.205	.210	.215	.220
$f(x)$	0.19739	0.20220	0.20699	0.21178	.21655

Group - C

Answer any *two* questions : 10×2=20

22. (a) Derive the Lagrange interpolation formula for $(n+1)$ unequally spaced tabular point.

(b) From the following compute the value of $f(1.2)$:

x	1.0	1.1	1.3	1.5
$f(x)$	0.36	0.32	0.26	0.21

5+5

23. (a) Solve the differential equation $\frac{dy}{dx} = x^2 y$ using Runge-Kutta Method to find $y\,(0.4)$ with $y(0) = 2$ taking $h = 0.1$. Do four iterations.

(b) Write the formula for Euler Modified Method to approximate the first order differential equation $\frac{dy}{dx} = f(x, y)$ with the initial condition $y\,(x_0) = y_0$.

8+2

24. Find the approximate value of $\int_0^1 \frac{dx}{1+x^2}$ correct up to four decimal places by Simpson's $\frac{1}{3}$ rule taking six subintervals. Hence, find the value of π, correct up to three decimal places. State the degree precision of this method. 7+2+1

25. (a) Solve the following system by LU factorization method :

$$3x + 4y + 2z = 15$$
$$5x + 2y + z = 18$$
$$2x + 3y + 2z = 10$$

(b) Why do we use Power Method? Is it an iterative method? 8+(1+1)

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B.Sc./5th Sem/MTM/25(NEP)

2025

5th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MI 5-T

(Multidisciplinary Studies Minor)

[Multivariate Calculus]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : 2×10=20

1. Show that $f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0) \\ 0, & \text{if } (x,y)=(0,0) \end{cases}$

is continuous at (0, 0).

2. Show that $f(x,y)=|x|+|y|$ is not differentiable at (0, 0).

3. Verify whether $f(x,y)=\begin{cases} \dfrac{x^2y^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0) \\ 0, & \text{if } (x,y)=(0,0) \end{cases}$

satisfies the conditions of Schwarz's theorem.

4. If $z=x^3-xy+y^3$, $x=r\cos\theta$, $y=r\sin\theta$, find z_r and z_θ.

5. Show that $\iint_E (x^2+y^2)dxdy=\dfrac{6}{35}$, where the region E is bounded by the parabolas $y=x^2$ and $x=y^2$.

6. Find a unit normal to the surface $x^2y+2xz=4$ at the point (2, –2, 3).

7. Evaluate $\int_1^2 \vec{r}\times\dfrac{d^2\vec{r}}{dt^2}dt$, given that $\vec{r}=t\hat{i}+t^2\hat{j}+t^3\hat{k}$.

8. Show that the vector $\vec{\nabla}\phi$ is perpendicular to the surface $\phi(x,y,z)=c$, where c is a constant.

9. Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field.

10. Find the total work done in moving a particle in a force field $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$ along the curve $x=t^2+1, y=2t^2, z=t^3$, from t = 1 to 2.

11. Check the existence of $\lim\limits_{(x,y)\to(0,0)}\dfrac{|x|}{y^2}e^{-\frac{|x|}{y^2}}$, $x\geq 0$.

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12. A particle, acted on by constant forces $(5\hat{i}+2\hat{j}+\hat{k})$ and $(2\hat{i}-\hat{j}-3\hat{k})$ is displaced from the origin to the $(4\hat{i}+\hat{j}-3\hat{k})$. Find the total work done by the forces.

13. For the function $f(x,y)=x^2-y^3-x^2y+y$, check whether the point $\left(0,\frac{1}{\sqrt{3}}\right)$ is a critical point or saddle point on a point of extreama.

14. Find $\oint_\Gamma(-4y\,dx+6x\,dy)$ where Γ is the circle defined by $\Gamma: x^2+y^2=1$.

15. If $\vec{F}-(ax+y+a)\hat{i}+\hat{j}-(x+y)\hat{k}$, where '$a$' is a constant and $\vec{F}.\,curl\,\vec{F}=0$, then find the value of a.

Group - B

Answer any ***four*** questions : 5×4=20

16. If $u=f(x,y)$ where $x=r\cos\theta$ and $y=r\sin\theta$, then show that $u_{xx}+u_{yy}=u_{rr}+r^{-2}u_{\theta\theta}+r^{-1}u_r$.

17. Show that $\iiint_R \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}=\frac{\pi^2}{8}$, where R is the region inside the sphere $x^2+y^2+z^2=1$ and in the first octant.

18. Verify Green's theorem in the plane for $\oint_C(3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region enclosed by $y=\sqrt{x}$ and $y=x^2$.

19. If $\vec{F}$ and $\vec{G}$ are two differentiable vector functions then prove that

$$\vec{\nabla}\times\left(\vec{F}\times\vec{G}\right)=\vec{F}(\vec{\nabla}\cdot\vec{G})-\vec{F}\cdot\vec{\nabla}\vec{G}+\vec{G}\cdot\vec{\nabla}\vec{F}-\vec{G}(\vec{\nabla}\cdot\vec{F}).$$

20. Find the minimum value of $x^2+y^2+z^2$ subject to the constraint $ax+by+cz=1$, where $a\neq0, b\neq0, c\neq0$.

21. If z is a function of two variables x & y and $x=c\cosh u\cos v$, $y=c\sinh u\sin v$ (c is a real no.), then show that

$$\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}=\frac{c^2}{2}(\cosh 2u-\cos 2v)\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right).$$

Group - C

Answer any ***two*** questions : 10×2=20

22. (a) Evaluate $\iint_S \vec{F}\cdot\hat{n}\,dS$, where $\vec{F}=x\hat{i}-y\hat{j}+(z^2-1)\hat{k}$, S is the surface of the region bounded by $x^2+y^2=4$, $z=0$, $z=4$ in the first octant.

P.T.O.

(b) Using Lagrange multiplier method, prove the inequality $\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$, $x \geq 0$, $y \geq 0$, $z \geq 0$. 5+5

23. (a) Show in a diagram the field of integration of the integral $\int_0^1 \left(\int_x^{\frac{1}{x}} \frac{y\,dy}{(1+xy)^2(1+y^2)} \right) dx$ and by changing the order of integration, show that the value of the integral is $\frac{\pi-1}{4}$.

(b) Evaluate by Stokes' theorem $\oint_C \sin z\,dx - \cos x\,dy + \sin y\,dz$, where C is the boundary of the rectangle :

$$0 \leq x \leq \pi,\ 0 \leq y \leq 1,\ z = 3.$$ 5+5

24. (a) When is a function $f(x, y)$ said to be differentiable at a point (x, y)? State the sufficient condition of differentiability of $f(x, y)$. Verify the sufficient condition for differentiability of the function :

$$f(x,y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \sin\frac{1}{y}, & x \neq 0,\ y \neq 0; \\ x^2 \sin\frac{1}{x}, & x \neq 0,\ y = 0; \\ y^2 \sin\frac{1}{y}, & x = 0,\ y \neq 0; \\ 0, & x = 0,\ y \neq 0; \end{cases}$$

(b) If $F(x,y)$ be a homogeneous function in x and y of degree n having continuous first order partial derivatives and $u(x,y) = (x^2+y^2)^{-\frac{n}{2}}$, then prove that $\frac{\partial}{\partial x}\left(F\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(F\frac{\partial u}{\partial y}\right) = 0$. (1+1+3)+5

25. (a) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orghogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$.

(b) What is the maximum directional derivative of $f(x,y) = y^2e^{2x}$ at $(2, -1)$ and in the direction of what unit vector does it occur? 5+5
