2025

1st Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MI 1-T (Minor)

(Calculus, Geometry and Ordinary Differential Equation)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** questions : 2×10=20

1. Show that the point of inflexion of the cuve $y = x^3 - 6x^2 + 12x - 5$ lies on the line $x + y = 3$.

2. Find the envelope of the family of straight lines

$$y = mx + \frac{a}{m}.$$

3. State the necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact.

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4. Find the length of the curve $y = \log \sec x$ between $x = 0$ and $x = \frac{\pi}{3}$.

5. Find the nature of the conic $3x^2 + 4xy + 2y^2 - 6x + 8y - 1 = 0$.

6. Find the polar equation of the straight line whose intercept on the initial line is a and which makes an angle α with it.

7. Show that the curve $y = x\log_e x (x > 0)$ is everywhere concave upwards.

8. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$, n being a positive integer greater than 1, then prove that $I_n = \frac{1}{n-1} - I_{n-2}$.

9. Find the radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0 = x - 2y + 2z - 3$.

10. What do you mean by asymptote? Does asymptote exist for every curve?

11. Find the value of $\lim_{x \to 1} x^{\frac{1}{x-1}}$.

12. Define point of inflexion of curve.

13. Find the value of b, if the solid generated by the revolution of the curve $xy = 1$, bounded by $y = 0$, $x = 2$, $x = b$ $(0 < b < 2)$.

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14. Find the equation of the right circular cone whose axis is $\frac{x}{1} = \frac{y}{0} = \frac{z}{-2}$ and radius equal to 7.

15. Find the integrating factor of the following differential equation $\frac{dy}{dx} + \frac{xy}{1-y^2} - y\sqrt{x} = 0$.

Group - B

Answer any ***four*** questions : 5×4=20

16. Prove that $\int_0^1 x^m (1-x)^n dx = \frac{m!n!}{(m+n+1)!}$ where m, n are positive integers.

17. Find the asymptotes of the curve $x^3 - 3xy^2 - 4x^2 + 6y^2 + 1 = 0$.

18. If $y = \sin(m\sin^{-1} x)$, then prove that $(1-x^2)y_2 - xy_1 + m^2 y = 0$.

Hence, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

19. Find the equation of cylinder generated by straight lines parallel to $\frac{x}{3} = \frac{y}{5} = \frac{z}{-4}$ and whose guiding curve is $3x^2 - 4y^2 = 5$, $z = 2$.

P.T.O.

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20. Find the solution of $\frac{dy}{dx} - y\tan x = \cos x$ by substitution $y = y_1 v(x)$ where $y_1 = \sec x$. 3+2

21. Let $P_n = D^n\left(x^n \log x\right)$.

Prove that $P_n = nP_{n-1} + (n-1)!$.

Hence show that $P_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}\right)$

Group - C

Answer any *two* questions : 10×2=20

22. (a) Reduce the quadratic form $[4x^2 + 9y^2 + 16z^2 + 12xy - 8yz - 16zx]$ to its canonical form.

(b) Solve the differential equation

$\left(x^2 + y^2\right)dx + 2xydy = 0.$ 5+5

23. (a) Find the equation of the right circular cone which passes through the coordinate axes and whose vertex is at the origin.

(b) Show that the general solution of $\frac{dy}{dx} + Py = Q$ can be written in the form

$y = Ce^{-\int Pdx} + e^{-\int Pdx}\int Qe^{\int Pdx}dx.$ 5+5

24. (a) Find the volume of the solid obtained by revolving one arc of a cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about x-axis.

(b) Find the area bounded by the curve $x = \cos t + 3$, $y = 4\sin t$. 5+5

25. (a) Prove that the locus of the foot of the perpendicular from a focus of the conic $\frac{l}{r} = 1 - e\cos\theta$ on a tangent to it is given by

$r^2\left(1 - e^2\right) - 2elr\cos\theta - l^2 = 0.$

(b) Show that the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upward for $-2 < x < 0$ and $2 < x < \infty$ but convex upwards for $-\infty < x < -2$, and $0 < x < 2$. Also show that $x = -2, 0, 2$ are points of inflexion. 5+(2+2+1)