2025

5th Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MDSE 1-T (Single Core Major Elective-I)

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[Linear Programming]

Group - A

Answer any *ten* questions : 2×10=20

1. A linear programming problem having 6 main constraints and 3 non-negativity constraints. Find the upper bound for the number of extreme points.

2. Show that the vectors (2,2,8), (1,0,4) and (1,2,4) are linearly dependent.

3. Show that the solution of a transportation problem is bounded.

4. In a game with 2×2 payoff matrix as given below with $a<d<b<c$, check whether there is a saddle point or not.

a	b
c	d

P.T.O.

5. Find the dual of the following L. P. P. :

Maximize $Z = 2x_1 + 3x_2 + x_3$

subject to

$4x_1 + 3x_2 + x_3 = 6;\ x_1 + 2x_2 + 5x_3 = 4;\ x_1 + 4x_2 \le 420,$

$x_1, x_2, x_3 \ge 0.$

6. Is (2, 3, 1) a basic solution of the equations? Justify.

$x_1 + x_2 + 2x_3 = 7;\ 3x_1 + 2x_2 + 5x_3 = 17;\ x_1, x_2, x_3 \ge 0.$

How many basic solutions this system has?

7. Distinguish between pure strategy and mixed strategy in a game.

8. Define zero-sum game.

9. Solve the games with the following payoff matrix :

$$\begin{pmatrix} 6 & -3 \\ -3 & 0 \end{pmatrix}$$

10. Define basic feasible solution.

11. Define convex polyhedron with example.

12. Justify the statement : "Assignment problem is an LPP".

13. Compare two-phase method and Big-M method.

14. What is the advantage to use duality to solve and LPP?

15. Define saddle point and optimal point of a game.

Group - B

Answer any ***four*** questions : 5×4=20

16. Use graphical method to solve the following LPP :

Maximize $Z = 50x_1 + 60x_2$

subject to $2x_1 + x_2 \le 300;\ 3x_1 + 4x_2 \le 504;$

$4x_1 + 7x_2 \le 812;\ x_1, x_2 \ge 0$

17. A company owns two flour mills, *A* and *B*, which have different production capacities, for high, medium and low grade flour. This company has entered a contract to supply flour to a firm every week with at lest 12, 8 and 24 quintals of high, medium and low grade respectively. It costs the company Rs. 1000 and Rs. 800 per day to run mill *A* and *B* respectively. On a day, mill *A* produces 6, 2 and 4 quintals of high, medium and low grade flour respectively. Mill *B* produces 2, 2 and 12 quintals of high, medium and low grade flour respectively. How many days per week should each mill be operated in order to meet the contract order most economically?

18. Prove that the set of all feasible solutions to a linear programming problem is a closed convex set.

19. Solve graphically the game whose pay-off matrix is

$$\begin{array}{c} \text{Player } B \\ \text{Player } A \begin{pmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{pmatrix}. \end{array}$$

P.T.O.

20. A hospital has the following requirements of nurses :

Period	Clock time (24 hours day)	Minimum number of nurses required
1	6A. M. - 10 A. M.	60
2	10 A. M. - 2 P. M.	70
3	2 P. M. - 6 P. M.	60
4	6 P. M. - 10 P. M.	50
5	10 P. M. - 2 A. M.	20
6	2 A. M. - 6 A. M.	30

Nurses report to the hospital wards at the beginning of each period and work for eight consecutive hours. The hospital wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate this as an L.P.P.

21. If x be any feasible solution to the primal problem

Maximize $Z = Cx$

subject to $Ax \leq 0, x \geq 0$

and v be any feasible solution to the dual problem, then prove that $Cx \leq b^T v$.

Group - C

Answer any ***two*** questions : 10×2=20

22. Solve the following LPP using Two-phase method :

Minimize $Z = 4x_1 + x_2$

subject to

$3x_1 + x_2 = 3;\ 4x_1 + 3x_3 \geq 6;\ x_1 + 2x_1 \leq 4;\ x_1, x_2 \geq 0.$ 10

23. (a) Solve the following travelling salesman problem :

	A	B	C	D	E
A	∞	2	4	7	1
B	5	∞	2	8	2
C	7	6	∞	4	6
D	10	3	5	∞	4
E	1	2	2	8	∞

(b) Put the following problem in standard form :

Maximize $Z = 3x_1 - 4x_2 - x_3$

subject to $x_1 + 3x_2 - 4x_3 \leq 12,$

$2x_1 - x_2 + x_3 \leq 20,$

$x_1 - 4x_2 - 5x_3 \geq 5,$

$x_1 \geq 0, x_2$ and x_3 are unrestricted in sign.

7+3

P.T.O.

24. (a) State and Prove Fundamental theorem of Gama Theory.

(b) Use dominance to reduce the Pay-off matrix and solve the following game problem given by the Pay-off matrix :

		A			
		B_1	B_2	B_3	B_4
B	A_1	1	7	2	7
	A_2	6	2	7	3
	A_3	1	1	6	2

6+4

25. (a) Using Vogel's approximation method, obtain an initial BFS to the balanced transportation problem given below :

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	34

(b) Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix :

	M_1	M_2	M_3	M_4
J_1	10	24	30	15
J_2	16	22	28	12
J_3	12	20	32	10
J_3	9	26	34	16

5+5

P.T.O.

OR

[Biomathamatics]

Group - A

Answer any ***ten*** questions : 2×10=20

1. Define the equilibrium point of a dynamical systems.

2. Define the Allee effect and explain its biological significance.

3. What is transcritical bifurcation and saddle node bifurcation?

4. State the Routh-Hurwitz stability criteria for a two-dimensional system.

5. Find the steady-states of $\frac{dx}{dt} = x - xy, \frac{dy}{dt} = xy - y$.

6. Solve the difference equation $N_{t+1} = rN_t$ for initial condition $N_0 = 5$ and $r = 2$.

7. Determine the nature of the bifurcation for the system $\frac{dy}{dt} = \mu y + y^2, \mu \in \mathbb{R}$.

8. For the Kermack-McKendrick SIR model, $S + I + R$ is equal to :

   (i) $2S$

   (ii) $3I$

   (iii) $4R$

   (iv) Constant (*Choose the correct option*)

9. A discrete logistic growth model is given by $N_{t+1} = rN_t(1 - N_t / K)$. For $r = 2.5, K = 100$, and $N_0 = 10$, compute N_1, N_2, N_3.

10. State the stability condition for a fixed point of a one-dimensional difference equation.

11. Determine the condition for which, the non-zero steady state of the logistic difference equation $x_{n+1} = rx_n(1 - x_n)$ is asymptotically stable.

12. Determine the maximum rate of growth of population size x corresponding to the logistic growth model $\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$.

13. If a population has exponential growth $\frac{dN}{dt} = rN$ with growth rate $r(> 0)$, then determine the time for which the population will be double.

14. Write the system of equations for the SI epidemic model and explain the meaning of each term.

15. Distinguish between SI and SIR epidemic models.

P.T.O.

Group - B

Answer any ***four*** questions : 5×4=20

16. (a) What do you mean by Hopf bifurcation?

    (b) Discuss supercritical bifurcation with example. 2+3

17. Derive the logistic growth solution $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$. For $r = 0.5$, $K = 1000$, and $N(0) = 10$, compute $N(t)$ at $t = 10$.

18. Describe the equilibrium points of Kermack-McKendrick SIR model. Compute basic reproduction number of the SIR model. Discuss the stability of equilibrium points. 3+1+1

19. Solve the decay model $C_{t+1} = (1-k)C_t$ for drug concentration with $C_0 = 100$ and $k = 0.1$. Plot values up to $t = 10$.

20. Show that the following system has stable limit cycle :

$$\begin{cases} \frac{dx}{dt} = -y + x(1 - x^2 - y^2) \\ \frac{dy}{dt} = x + y(1 - x^2 - y^2) \end{cases}$$

21. Linearize the classical Lotka-Volterra model,

$$\begin{cases} \frac{dx}{dt} = \alpha x - \beta xy \\ \frac{dy}{dt} = -\gamma y + \delta xy \end{cases}$$

about the positive steady state, where $\alpha, \beta, \gamma, \delta$ are positive parameters.

Group - C

Answer any ***two*** questions : 10×2=20

22. Derive and analyze the SIRS epidemic model :

$$\frac{dS}{dt} = -\beta SI + \gamma R, \frac{dI}{dt} = \beta SI - \delta I, \frac{dR}{dt} = \delta I - \gamma R$$

Find equilibrium points and discuss stability conditions.

23. Consider the discrete-time predator-prey system :

$$x_{n+1} = ax_n(1 - x_n) - bx_n y_n,$$
$$y_{n+1} = -cy_n + dx_n y_n$$

where a, b, c, d are parameters. Find the steady states of the system and discuss their stability.

24. Consider the planar system :

$$\begin{cases} \frac{dx}{dt} = \frac{x}{2021 + 2022 + 2023}(2021 - 2022x - 2023y) \\ \frac{dy}{dt} = \frac{y}{2024 + 2025 + 2026}(2024 - 2025x - 2026y) \end{cases}$$

(a) Find all equilibrium points of the system.

(b) Using the **Dulac-Bendixon criteria,** prove that system has no periodic solution in the positive quadrant $\Omega = \{(x, y) : x > 0, y > 0\}$. 3+7

25. Consider the system of ordinary differential equations :

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{ry}{1+ax} - kx - \dfrac{\alpha xy^2}{cyx+1}, \\ \dfrac{dy}{dt} = kx - \dfrac{\delta y}{1+bx}, \\ x(0) > 0, y(0) > 0, \end{cases}$$

where $x(t)$ represents eggs, $y(t)$ represents adult social insects, and $r, a, k, \alpha, c, \delta, b (> 0)$ are constants.

(a) Find all equilibrium points of the system.

(b) Linearize the system around a non-trivial equilibrium $(x^*, y^\wedge *)$ and write down the Jacobian matrix completely.

(c) Using the eigenvalues of the Jacobian, determine the conditions for local stability of the non-trival equilibrium. Discuss the biological interpretation of your result in terms of population persistence or collapse. 2+4+4

OR

[Industrial Mathematics]

Group - A

Answer any ***ten*** questions : 2×10=20

1. Define a forward problem with an example from medical imaging.

2. What is meant by regularization in inverse problems?

3. Write the mathematical expression of the Radon Transform in 2D.

4. State the Central Slice (Projection) Theorem.

5. What is Tikhonov regularization?

6. Define convolution and mention its role in image reconstruction.

7. What is meant by discretization in tomography?

8. Write the matrix form of a linear inverse problem.

9. What is noise amplification in inverse problems?

10. Define the attenuation coefficient and mention its physical unit.

11. What is the significance of eigenvalues in solving inverse problems?

12. What is meant by back-projection without filtering?

P.T.O.

13. Mention one limitation of Algebraic Reconstruction Techniques (ART).

14. What is a sinogram?

15. Write one application of inverse problems in non-destructive testing.

Group - B

Answer any ***four*** questions : 5×4=20

16. Derive the expression for the Radom Transform of the function $f(x,y) = x^2 + y^2$ at $\theta = 0^{\circ}$.

17. Show that convolution in spatial domain corresponds to multiplication in frequency domain using Fourier Transform.

18. Explain the concept of regularization parameter. How does it control stability in ill-posed problems?

19. Formulate a simple 3×3 image reconstruction problm as a system of linear equations and explain how it can be solved using matrix methos.

20. Discuss the difference between analytical reconstruction (FBP) and iterative reconstruction methods.

21. A beam with intensity $I_0 = 200$ reduces to 120 after passing through 5 cm of material :

    (a) Find the attenuation coefficient.

    (b) Determine the half-value thickness.

Group - C

Answer any ***two*** questions : 10×2=20

22. Explain the mathematical formulation of CT image reconstruction using the Projection Slice Theorem. Derive how filtered back-projection is obtained from it.

23. Consider the inverse problem $Ax = b$, where

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 7 \\ 6 \\ 4 \end{bmatrix}$$

    (a) Interpret this system as a tomographic reconstruction problem.

    (b) Perform one interation of ART starting from zero initial guess.

24. Discuss the role of Singular Value Decomposition (SVD) in solving ill-conditioned inverse problems. Explain how small singular values affect stability.

25. Describe a real industrial application of inverse problems (e.g., thermal imaging, seismic exploration, or electrical impedance tomography). Present the mathematical model and discuss solution startegies.

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