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PG/1st Sem /PHS/25

2025

M.Sc. 1st Semester Examination

PHYSICS

Paper : PHSC404X0

[Quantum Mechanics-I]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***four*** questions : 2×4=8

1. For the angular momentum operators $\hat{J}_i$, show that $\left[\hat{J}_+, \hat{J}_-\right] = 2\hbar\hat{J}_z$, where $\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$.

2. Derive the expression for position space wave function $\langle x | \alpha \rangle = \psi_\alpha(x)$ as an integral over the momentum space wave function $\langle p | \alpha \rangle = \phi_\alpha(p)$ starting from the state $|\alpha\rangle$.

3. Show that Clebsch-Gordan coefficients vanish unless $m = m_1 + m_2$, where the symbols have their usual meanings.

P.T.O.

4. For a one dimensional harmonic oscillator, using the ladder operators $\hat{a}$ and $\hat{a}^\dagger$, show that the eigenvalues of the number operator $\hat{N} = \hat{a}^\dagger \hat{a}$ are non-negative integers.

5. Show that if a system possesses a symmetry, then it leads to degeneracy of energy eigenvalues.

6. Write down the matrix representation of the spin –½ operator, $\hat{S}_x$ and find its normalized eigenvectors in the $|\pm, z\rangle$ basis. Further write down the unitary matrix U such that $\hat{U}|+,z\rangle = |+,x\rangle$.

Group - B

Answer any ***four*** questions : 4×4=16

7. For a one dimensional harmonic oscillator obtain the matrix representations of the operators $\hat{X}$ and $\hat{P}$. Verify that $\left[\hat{X}, \hat{P}\right] = i\hbar$.

8. Consider a two-state system with orthonormal basis states $\{|1\rangle, |2\rangle\}$. The Hamiltonian operator for the system is given by $\hat{H} = \epsilon\left(|1\rangle\langle 1| + |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$ where ϵ is a constant having the dimension of energy. Suppose it is known that at time $t = 0$ the system is in state $|\psi(0)\rangle = |1\rangle$. Find $|\psi(t)\rangle$.

9. Given that $\langle x|\hat{P}_x|\alpha\rangle = -i\hbar\frac{\partial}{\partial x}\langle x|\alpha\rangle$, where $|\alpha\rangle$ is any general state. Show that $\langle x | p_x\rangle = (2\pi\hbar)^{-1/2} e^{ipx/\hbar}$.

10. Two different spin $\frac{1}{2}$ particles have a Hamiltonian $\hat{H} = \alpha + \beta \hat{S}_1 . \hat{S}_2 + \gamma(\hat{S}_{1z} + \hat{S}_{2z})$ where α, β and γ are constants. Find the energy eigenvalues and the normalized eigenvectors of the system.

11. Find the matrix representation of the angular momentum operator $\hat{J}_y$ for $j = 1$ in the $|j, m\rangle$ basis such that the $\hat{J}_z |j, m\rangle = m\hbar |j, m\rangle$.

12. Consider a state $|\psi\rangle$ and a Hamiltonian $\hat{H}$ whose matrix representations are given as

$$|\psi\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad [\hat{H}] = \frac{E_0}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

where E_0 is a constant. If energy is measured, what values would we obtain and with what probabilities?

Group - C

Answer any *two* questions : 8×2=16

13. (a) The Hamiltonian operator for a spin-1/2 system is given by $\hat{H} = \omega \hat{S}_z$ where ω is a real constant. The state of the system at $t = 0$ is $|+, x\rangle$ find the expectation values of the operators $\hat{S}_x, \hat{S}_y, \hat{S}_z$ as a function of time using the Heisenberg picture.

P.T.O.

(b) Consider a one-dimensional harmonic oscillator with the ground state wave function $\psi_0(x) = e^{-x^2/(2x_0^2)}$, where x_0 is a constant. Find the value of the coefficient α so that $\psi_2(x) = (1+\alpha x^2)e^{-x^2/(2x_0^2)}$ is orthogonal to $\psi_0(x)$. 5+3

14. If the eigenvalues of $\hat{J}^2$ and $\hat{J}_z$ are given by the eigenvalue equations

$\hat{J}^2 |\lambda, m\rangle = \lambda\hbar^2 |\lambda, m\rangle$ and $\hat{J}_z |\lambda, m\rangle = m\hbar |\lambda, m\rangle$, then

show that (a) $\lambda \geq m^2$ (b) $\lambda = j(j+1)$ and (c) $-j \leq m \leq j$, where the symbols have their usual meanings. 3+2+3

15. (a) Consider the angular momentum addition of two spin-half particles. Compute all the associated Clebsch-Gordan coefficients and rewrite all the states in the $|j_1, j_2; j, m\rangle$ basis in terms of those in $|j_1, j_2; m_1, m_2\rangle$ basis. You must show all steps involved in the computation.

(b) If $|\alpha\rangle$ is an eigenstate of the parity operator $\hat{\pi}$ show that the corresponding eigenfunction $\psi_\alpha(x)$ is either an even or an odd function of x.

6+2

16. (a) Consider a one-dimensional potential that is invariant under lattice translation such that $\hat{\tau}(a)^{\dagger}V(x)\hat{\tau}(a) = V(x+a) = V(x)$. Show that the state $|\theta\rangle$ defined as $|\theta\rangle \sum_{n=-\infty}^{\infty} e^{in\theta}|n\rangle$ is an eigenstate of the lattice translation operator $\hat{\tau}(a)$. Further using the tightbinding approximation, find the energy eigenvalues as a function of θ.

(b) A spin-half system has a Hamiltonian $H = \omega\hat{S}_z$. If the system is in the state $|+,x\rangle$ at time $t = 0$. Find the probabilities of finding the system in the states $|\pm,x\rangle$ as function of time. 4+4

Internal Assessment : 10 marks
