PG/1st Sem /PHS/25

2025

M.Sc. 1st Semester Examination

PHYSICS

Paper : PHSC402X0

[Classical Mechanics]

Full Marks : 50

Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***four*** questions : 2×4=8

1. The Hamiltonian of mass m is given by $H = \frac{(p-\alpha q)^2}{2m}$, where α is a non-zero constant. Find the value of $\ddot{q}$.

2. A particle moves in two dimensions under the potential $V(x, y) = 2x - 3y$. Find the value of $\frac{d}{dt}\left(3p_x + 2p_y\right)$.

3. Explain briefly how the Poisson bracket of a dynamical variable with the Hamiltonian is related to the conservation of that dynamical variable.

P.T.O.

4. Show that the transformation $Q = \sqrt{2q}\, e^{\alpha} \cos p$ and $Q = \sqrt{2q}\, e^{-\alpha} \sin p$ is canonical.

5. What are action-angle variables?

6. Briefly explain the Noether's theorem connecting the symmetry and conservation law of a system.

Group - B

Answer any ***four*** questions : 4×4=16

7. Consider a charged particle of mass m and charge e in a uniform magnetic field $B = B\hat{z}$, with vector potential given by the Landau gauge $A = (0, Bx, 0)$. The Hamiltonian for this system is

$$H = \frac{1}{2m}\left[p_x^2 + \left(p_y - eBx\right)^2 + p_z^2\right].$$

Define the quantity $Q = p_x - eBy$. Show that Q is a constant of motion using Hamilton's equations.

8. Using the Hamilton's principle, derive the equation of a frictionless path that will need the shortest time to reach one fixed point from another, in the presence of gravity.

9. The Lagrangian of motion of a two-dimensional isotropic harmonic oscillator is given by

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}K\left(x^2 + y^2\right).$$

Given $F = \dot{x}\dot{y} + \alpha xy$ is a constant of motion. Find α.

10. For a symmetric top fixed at one point, illustrate the concepts of Euler's angles with a suitable diagram. Write down the expression of ω in terms of these angles.

3+1

11. Derive Lagrange's equation of motion from Hamilton's principle.

12. Explain the concept of a Poincaré map and discuss its usefulness in the study of chaotic systems.

Group - C

Answer any *two* questions : 8×2=16

13. Consider a uniform magnetic field $B = B\hat{z}$ with the vector potential $A = (0, Bx, 0)$ and scalar potential $\varphi = 0$.

(a) Write the Lagrangian L of the system and derive the Lagrange equations of motion.

(b) Using the given cyclotron frequency $\omega_c = {qB}/{m}$, solve the particle's motion and provide explicit expressions for $x(t)$ and $y(t)$ for the initial conditions $r(0) = (x_0, y_0)$ and $\dot{r}(0) = (v_{x0}, v_{y0})$.

3+5

P.T.O.

14. (a) Give an example for a system (with brief explanation) where

    (i) the Hamiltonian is conserved but is not the total energy of the system and

    (ii) the Hamiltonian is the total energy of the system but is not conserved.

    (b) Write down the Hamilton-Jacobi equation for a generating function and, explain briefly the concept of Hamilton's principle function. (2+2)+(2+2)

15. What is Hamilton's principal function? Consider a one-dimensional system of mass m, whose Lagrangian is $L = \frac{1}{2}m\dot{x}^2 + \lambda\dot{x}$, where, λ is a constant. Obtain the canonical momentum p, construct the Hamiltonian $H(x, p)$. Write down the Hamilton-Jacobi equation. Solve the Hamilton-Jacobi equation and obtain the equation of motion. 1+1+1+1+4

16. Write down the three Euler angles and briefly state their physical significance. Derive the Euler equations of motion for a rigid body and mention one physical situation where they are applicable. 3+5

Internal Assessment : 10 marks
