

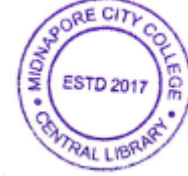
2025

## M.Sc. 1st Semester Examination

## PHYSICS

Paper : PHSC401X0

[Mathematical Physics]



Full Marks : 50

Time : Two Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

## Group - A

Answer any *four* questions :  $2 \times 4 = 8$ 

1. If  $A = \begin{pmatrix} -1 & 4 \\ 2 & 1 \end{pmatrix}$ , then prove that  $3 \tan A = A \tan 3$ .

~~2.~~ If the trace of an  $n \times n$  idempotent matrix is  $n - 1$ , then find the value of  $\det A$ .

3. Find Fourier sine transform of  $\frac{1}{x}$ .

4. Find the value of  $\epsilon_{ikm}^{ikm}$ .

P.T.O.



( 2 )

5. Show that there does not exist any isomorphism from Klein's 4-group  $V_4$  to the cyclic group  $C_4$  of order 4.
6. Prove that  $3 \otimes 3 = 6 \oplus \bar{3}$ .

### Group - B

Answer any *four* questions :

4×4=16

7. Solve by Lagrange's method :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x.$$

8. If  $A = \begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix}$ ; find  $4^A$ .

9. If the Fourier sine transform of  $f(x)$  is

$$\frac{1 - \cos n\pi}{n^2 \pi^2}; \quad (0 \leq x \leq \pi). \text{ Find } f(x).$$

10. Prove that a group of order 3 is always a cyclic group.

11. Solve  $L \frac{di}{dt} + \frac{q}{c} = E_0 \delta(t); i(0) = 0.$

Solve using Laplace transform.

12. Prove that  $A = \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}$  and  $B = \begin{pmatrix} -xy & x^2 \\ -y^2 & xy \end{pmatrix}$

are tensors.



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**Group - C**

Answer any *two* questions : 8×2=16

13. (a) Prove that  $A_{,v}^v = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x_v} \{ \sqrt{g} A^v \}$

where  $g = \det(g_{\mu\nu})$

(b) Evaluate  $\hat{L} \left\{ e^{-t} \int_0^t \frac{\sin t}{t} dt \right\}$  4+4

14. Solve  $f(t) = 1 + 2 \int_0^t f(t-x) e^{-2x} dx$  and verify your solution.

15. (a) Find the Green's function of the differential

equation  $\frac{d^2 \psi(x)}{dx^2} + \psi(x) = f(x)$

with boundary conditions  $\psi(0) = \psi(1) = 0$ .

(b) Show that  $x \frac{d}{dx} \delta(x) = -\delta(x)$ . 5+3

16. Find the invariant subgroup and factor group of equilateral triangle ( $D_3$ ). 4+4

**Internal Assessment : 10 marks**

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