2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTME404A0 / B1 & B2

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meanings.

Paper : MTME404A0

[Ordinary Differential Equation and Special Functions]

Group - A

Answer any ***four*** questions : 2×4=8

1. Prove $F(a,b,c;z)=(1-z)^{-a}F\left(a,c-b,c;\frac{z}{z-1}\right)$. [CO4]

2. Let $P_n(z)$ be the Legendre polynomial of dgree n such that $P_n(1)=1,\ n=1,2,3,...$ Find the value of $\int_{-1}^{1}(P_3(z))^2\,dz.$ [CO4]

P.T.O.

3. Define self-adjoint differential equation with an example. [CO1]

4. For what value of n the general solution of Bessel's differential equation will be of the form

$$W = AJ_n(z) + BJ_{-n}(z).$$ [CO4]

5. Green's function of the differential operator L of the non-homogeneous differential equation : $Lu(x) = f(x)$. [CO2]

6. What is the role of the weight function in a Sturm-Liouville problem? [CO1]

Group - B

Answer any ***four*** questions : 4×4=16

7. Prove that, $P_{2m}(0) = (-1)^m \frac{(2m)!}{2^{2m}(m!)^2}$. [CO4]

8. Prove that the zeros of Legendre polynomial are all real, distinct and lie between –1 and 1. [CO4]

9. Express $z^4 + 2z^3 + 2z^2 - z - 3$ in terms of Legendre polynomial. [CO4]

10. Prove that, all the eigenvalues of a regular Strum-Liouville system with $r(x) > 0$, are real. [CO1]

11. Show that $J_0^2(z) + 2\sum_{n=1}^{\infty} J_n^2(z) = 1$ and prove that for real z, $|J_0(z)| < 1$, and $|J_n(z)| < \frac{1}{\sqrt{2}}$, for all $n \geq 1$. [CO4]

12. If the vector functions $\varphi_1, \varphi_2, \ldots, \varphi_n$ defined as follows:

$$\varphi_1 = \begin{bmatrix} \varphi_{11} \\ \varphi_{21} \\ \cdot \\ \varphi_{n1} \end{bmatrix}, \varphi_2 = \begin{bmatrix} \varphi_{12} \\ \varphi_{22} \\ \cdot \\ \varphi_{n2} \end{bmatrix}, \ldots \varphi_n = \begin{bmatrix} \varphi_{1n} \\ \varphi_{2n} \\ \cdot \\ \varphi_{nn} \end{bmatrix}$$

be n solutions of the homogeneous linear differential equation $\frac{dx}{dt} = A(t)x(t)$ in the interval $a \leq t \leq b$, then these n solutions are linearly dependent in $a \leq t \leq b$ iff Wronskian $W[\varphi_1, \varphi_2, \ldots, \varphi_n] = 0 \forall t$, on $a \leq t \leq b$. [CO3]

Group - C

Answer any ***two*** questions : 8×2=16

13. (i) Prove that if $f(z)$ is continuous and has continuous derivatives in [–1, 1] then $f(z)$ has unique Legendre series expansion is given by $f(z) = \sum_{n=0}^{\infty} C_n P_n(z)$ where P_n's are Legendre polynomials $C_{n} = \frac{2n+1}{2} \int_{-1}^{1} f(z) P_n(z), n = 1, 2, 3 \ldots$ [CO4]

(ii) Express $J_4(z)$ in terms of $J_0(z)$ and $J_1(z)$.

5+3 [CO4]

P.T.O.

14. (i) Using Green's function method, solve the following differential equation $\frac{d^2y}{dx^2} - y = -2e^x$ with boundary conditions, $y(0) = y'(1)$, $y(l) + y'(l) = 0$. [CO2]

(ii) Prove that $J_n(z) = 0$ has no repeated roots except at $z = 0$, where $J_n(z)$ is the Bessel's function. 5+3 [CO4]

15. (i) Find the characteristics values and characteristic functions of the Sturm-Liouville problem $(xy')' + \frac{\lambda y}{x} = 0;\ y'(1) = 0,\ y(b) = 0,\ b > 1.$ [CO1]

(ii) Let, $P_n(z)$ be the Legendre polynomial of degree $n \geq 0$. If $1 + z^{10} = \sum_{n=0}^{10} C_n P_n(z)$, then find the value of C_5. 5+3 [CO4]

16. (i) Deduce the integral formula for confluent hypergeometric function. [CO4]

(ii) Find the general solution of the non-homogeneous system,

$$\frac{dX}{dt} = \begin{pmatrix} 7 & 4 & 4 \\ -6 & -4 & -7 \\ -2 & -1 & 2 \end{pmatrix} X + \begin{pmatrix} 2t+3 \\ -3 \\ 3t-9 \end{pmatrix}, \text{ where } X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

2+6 [CO5]

Internal Assessment : 10 marks

Paper : MTME404B1/B2

(Data Science and Graph Theory)

MTME404B1

(Unit-I : Data Science)

Group - A

Answer any *two* questions : 2×2=4

1. What is meant by Euclidean Norm? Write its mathematical expression. Explain with example. [CO1]

2. What is topology of data? [CO1]

3. How spread of a data set can be measured? Explain with examples. [CO2]

4. What is a Box Plot? Mention any two components of a box plot. [CO2]

Group - B

Answer any *two* questions : 4×2=8

5. Suppose, you have a list of marks obtained in Mathematics, Physics, Chemistry and Biology for one hundred students in a school. Write a Python program to find the average result considering (a) all subjects, (b) any subject for all data and some missing data using two dimensional NumPy array. [CO2]

P.T.O.

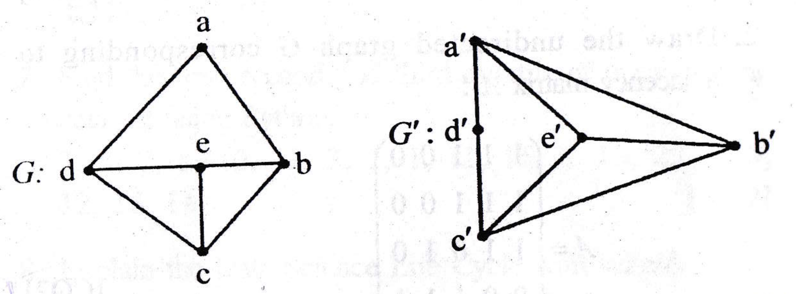
Group - B

Answer any ***two*** questions : 4×2=8

5. If G is a simple connected planar graph with $n(\geq 3)$ vertices and e edges, then prove that $e \leq 3n-6$ and using this, verify that the complete graph, K_5 is planar or not. [CO1]

6. In a connected graph G, prove that any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set. [CO2]

7. Show that the graphs G and G' are isomorphic :



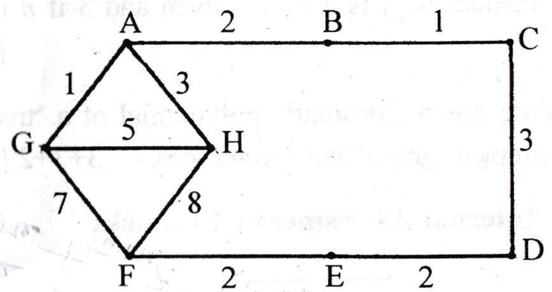
[CO1]

8. Define the term "Chromatic number" for the graph colouring with example. Find the chromatic number of the Wheel graph W_6. [CO2]

Group - C

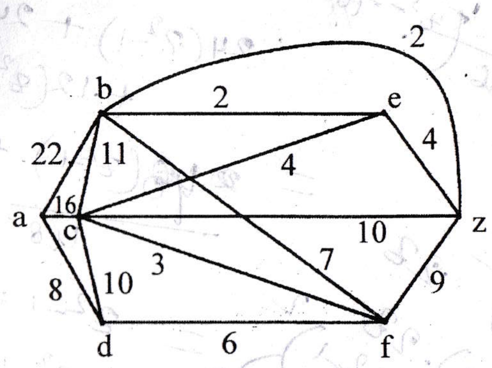
Answer any ***one*** question : 8×1=8

9. (i) Solve the travelling salesman problem for the following weighted graph :



[CO2]

(ii) Apply Dijkstra's algorithm to the graph given below and find the shortest path from a to z :



3+5 [CO2]

P.T.O.

10. (i) Prove that the relation $\chi(G) \leq \Delta(G)+1$ for any graph G where $\chi(G)$ is the chromatic number and $\Delta(G)$ is the maximum degree of a vertex in G. [CO2]

(ii) Show that the chromatic number of a cycle with n vertices (C_n) is 2 if n is even and 3 if n is odd. [CO2]

(iii) Write down Chromatic polynomial of a 'tree' and 'complete graph' with n vertices. 3+3+2 [CO2]

Internal Assessment : 10 marks