

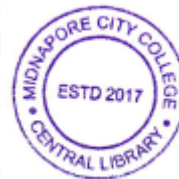
2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTMC402X2

[Abstract Algebra]



Full Marks : 25

Time : One Hour

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.
Notations and symbols have their usual meanings.*

Group - A

Answer any *two* questions : $2 \times 2 = 4$

1. Give an example of a Principal ideal domain which is not Euclidean domain. [CO1]
2. Let $f : G \rightarrow H$ be a homomorphism of groups. Show that $f(G') \subseteq H'$. [CO1]
3. Show with example that finite extensions need not be simple. [CO2]
4. Let D be a Euclidean domain with a Euclidean valuation v . If $a | b$ and $v(a) = v(b)$, then a and b are associates in D . [CO1]

P.T.O.

(2)

Group - B

Answer any *two* questions : 4×2=8

5. Show that the group A_n is generated by 3-cycles. Also show that all 3-cycles are conjugates in A_n if $n \geq 5$. [CO2]

6. Prove or disprove that the domain $D = \mathbb{Z}[\sqrt{-5}]$ is a UFD. [CO1]

7. If G is solvable and $H \triangleleft G$, show that G/H is solvable. [CO2]

8. If K is algebraic over F and L is algebraic over K , then show that L is algebraic over F . [CO2]

Group - C

Answer any *one* question : 8×1=8

9. (i) Show that if R is a principal ideal domain, then it is a unique factorization domain. Give an example to show that the converse is not true. [CO1]

(ii) Prove that subgroups of a solvable group are solvable. 6+2 [CO2]

10. (i) Is the regular pentagon constructible by using ruler and compass only? [CO1]

(ii) Prove that $\sqrt[3]{2}$ is not ruler and compass constructible. [CO2]

(3)

(iii) Find a gcd of the elements $1 + 3i$ and $3 + 4i$ in $\mathbb{Z}[i]$. 4+2+2 [CO1]

Internal Assessment : 5 marks

