2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTMC402X1

[Classical Mechanics]

Full Marks : 25 Time : One Hour

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meanings.

Group - A

Answer any *two* questions : 2×2=4

1. Find the expression for the generalised force in terms of generalised coordinates. [CO1]

2. If $Q = ap + bq$, $P = cp + dq$ is a canonical transformation, then find the relation among a,b,c,d. [CO2]

3. "*The Poisson bracket of two constants of motion is itself a constant of motion.*" — Explain it. [CO2]

4. Set up the Hamiltonian for a system whose Lagrangian is given by $L(x,\dot{x}) = \frac{1}{2}x^2 - \frac{1}{2}w^2\dot{x}^2 - ax^3 + \beta x\dot{x}^2$, where α, β and w are constants. [CO1]

P.T.O.

Group - B

Answer any ***two*** questions : 4×2=8

5. State and prove the Poincaré theorem. [CO2]

6. If the equations of transformation do not depend explicitly on time and the potential energy is velocity independent, then prove that H is the total energy of the system. [CO1]

7. Prove that $J = \int_{x_0}^{x_1} F(y, y', x)\,dx$

   will be stationary only if $\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0.$ [CO1]

8. Let $G = G_1(q_1, q_2, ..., q_n, Q_1, Q_2, ..., Q_n, t)$ be a generating function of a canonical transformation. Prove that $p_j = \frac{\partial G_1}{\partial q_j}$, $P_j = -\frac{\partial G_1}{\partial Q_j}$ for all j. Hence, prove that if the canonical transformation is given, then one can determine the generating function. [CO2]

Group - C

Answer any ***one*** question : 8×1=8

9. Suppose a particle of mass m_0 is moving with a velocity v, then show that its mass at any time is given by

   $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$, where c is the speed of light. [CO2]

10. State the principle of least action. Deduce Hamilton's equations of motion from this principle. [CO1]

Internal Assessment : 5 marks
