2025

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS

Paper : MTMC401X2

[Complex Analysis]

Full Marks : 25 Time : One Hour

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meanings.

Group - A

Answer any ***two*** questions : 2×2=4

1. With necessary conditions, write the Homotopy form of Cauchy's theorem. [CO1]

2. Using the argument principle, evaluate $\int_C \frac{f'(z)}{f(z)}dz$ when $f(z)=\frac{(z^2+1)^2}{\left(z^2+3z+2\right)^3}$ and $C:|z|=2$, taken positive sense. [CO2]

P.T.O.

3. Show that the function $e^{-\frac{1}{z^2}}$ has no singularities. [CO1]

4. Find Branch point (if any) of $f(z)=\frac{\sin\sqrt{z}}{\sqrt{z}}$. [CO1]

Group - B

Answer any ***two*** questions : 4×2=8

5. (i) Classify the singularity $z = 0$ of the function $f(z)=\frac{\cosh(z^3)-1}{z^7}$ in terms of removable, pole and essential singularity. In case $z = 0$ is a pole, specify the order of the pole. [CO1]

(ii) With the help of residue theorem, evaluate $\int_C \frac{\cosh(z^3)-1}{z^7}dz$, where $C:|z|=1$ taken in the positive direction. 4+4 [CO1]

6. Using the principle of residue, evaluate $\int_0^{\pi}\frac{d\theta}{a+\cos\theta}, a>1$. [CO2]

7. State and prove Casorati-Weierstrass's theorem. [CO2]

8. Prove that all the roots of the equation $z^5-12z^2+14=0$ lie between the circles $|z|=1$ and $|z|=\frac{5}{2}$. How many roots lie inside $|z|=2$. [CO2]

Group - C

Answer any ***one*** question : 8×1=8

9. (i) Show that if an analytic function $f(z)$ has a pole of order m at $z = a$, then, $\frac{1}{f(z)}$ has a zero of order m at $z = a$.

(ii) Using the calculus of residue evaluate

$$\int_0^{\infty}\frac{2x^2-1}{x^4+5x^2+4}dx.$$ 2+6 [CO2]

10. (i) What is the winding number? Explain it briefly.

(ii) If $f(z)$ behaves as a meromorphic function inside C, also analytic and non-zero on C then prove that

$$\int_C\frac{f'(z)}{f(z)}dz = 2\pi i\times(\text{winding number}).$$

2+6 [CO2]

Internal Assessment : 5 marks