2023

BCA 3rd Semester (CBCS) Examination

Discrete Mathematics

PAPER — CC-7T

Full Marks : 80

Time : 3 hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer from **all** the Groups as directed.

GROUP—A

1. Answer *any* **ten** questions : 2×10=20

(a) If $A=\{3,5,7,8,9\}$, $B=\{1,5,6,8,10\}$ and $C=\{2,3,4,7,8\}$, then find $(A-B)\cup(B-C)$.

(b) Let R be the following relation on

$A = \{1, 2, 3, 4\}$; $R = \{(1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$

Then find the domain and range of R.

(c) Let $f(x) = x^2 + x$ and $g(x) = x + 1$, then find $g \circ f(x)$.

(d) Find the number of relation from $A = \{1, 2, 3\}$ to $B = \{a, b\}$.

(e) Let p be "He is writing a story" and q be "He is playing football". Give a simple verbal sentence which describes the statement $q \vee \neg p$.

(f) Show that $p \vee (p \rightarrow q)$ is a tautology.

(g) If ${}^{n}C_{12} = {}^{n}C_{8}$, then find the value of n.

(h) Three persons enter a railway compartment. If there are 5 seats vacant, in how many ways can they take these seats?

(i) What is the minimum number of edges in a connected graph with 97 vertices?

(j) Can a simple graph exist with 9 vertices, each of degree 3? Justify.

(k) Consider the set Q of rational numbers and let $*$ be the operation on Q defined by

$$a * b = a + b - ab$$

Find $7 * \left(-\frac{1}{2}\right)$.

(l) Let $S = N \times N$. Let $*$ be the operation on S defined by $(a, b) * (a', b') = (aa', bb')$. Examine whether $*$ is associative.

(m) What is an AND gate?

(n) Draw a 3-regular graph.

(o) Define the algebraic system $(R, +, \cdot)$ as a ring.

GROUP—B

Answer *any* **six** questions : 5×6=30

2. Among 75 children who went to an amusement part, where they would ride on merry-go-round roller coaster and ferris wheel. It is known that, 20 of them had taken all three rides and 55 had taken at least two of the three rides. Each ride costs ₹0·50 and total receipt of park is ₹70. Determine the number of children who did not try any of the rides.

3. Solve the recurrence relation

$$a_r - 7a_{r-1} + 10a_{r-2} = 0 \text{ for } r \geq 2$$

4. Simplify the Boolean expression and construct a network using logic gates for the expression

$$z = \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

5. Draw the digraph of the incidence matrix

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

6. Show that the set $G = \{1, 2, 3, 4, 5, 6\}$ forms a commutative group with respect to the operation multiplication modulo 7.

7. In how many ways 5 white balls and 3 black balls can be arranged in a row so that no two black balls may be together?

8. Obtain the principal conjunctive normal form of $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$.

9. Show by mathematical induction that $8^n - 3^n$ is a multiple of 5 for $n \geq 1$.

(Continued)

GROUP—C

Answer *any* **three** questions : 10×3=30

10. *(a)* Show that the function defined by $f : z \rightarrow z$ given by $f(x) = 2x + 3$ for all $x \in z$ is injective but not surjective.

(b) Define Hasse diagram. Draw the Hasse diagram representing the positive divisors of 60. 5+5=10

11. *(a)* Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

(b) Use a Karnaugh map to find a minimal sum-of-products form for

$$E = xy' + xyz + x'y'z' + x'yzt'$$

5+5=10

12. *(a)* Solve the congruence equation

$$f(x) = 4x^4 - 3x^3 + 2x^2 + 5x - 4 \equiv 0 \pmod 6$$

(b) Prove that a finite connected graph G is Eulerian if and only if each vector has even degree. 5+5=10

(Turn Over)

13. *(a)* Prove that ${}^{17}C_6 = {}^{16}C_5 + {}^{16}C_6$

(b) Draw diagraph for relation R on

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Let xRy whenever y is divisible by x. Is R partial ordering? 5+5=10

14. *(a)* Show that z_8, the additive group of all integers modulo 8 is a cyclic group. Find all generators of z_8.

(b) Define the term 'sub-lattice'. Give an example. 5+5=10

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