2023

5th Semester Examination

PHYSICS (Honours)

Paper : DSE 1-T

[CBCS]

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

[Advanced Mathematical Physics-I]

Full Marks : 40 Time : Two Hours

Group - A

Answer any ***five*** questions out of following ***eight***. 2×5=10

1. Find the Laplace transform of 1st derivative of $\frac{df}{dt}$

   i.e. $L\left[\frac{df}{dt}\right] = ?$

2. If A^i and B_j are components of a contravariant and covariant tensor of rank one, then show that $C^i_j = A^i B_j$ are the components of a mixed tensor of rank 2.

3. Define Alternate tensor and Kronecker tensor.

P.T.O.

4. Using tensor index notation show that

$$\vec{\nabla}.\left(\vec{A}\times\vec{B}\right)=\vec{B}.\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\left(\vec{\nabla}\times\vec{B}\right)$$

5. Show that $N \times N$ singular matrices do not form vector space of dimension N^2.

6. If the vectors $\vec{u}$ and $\vec{v}$ are given by $\vec{u}=\sum_i u_i\hat{e}_i$ and $\vec{v}=\sum_i v_i\hat{f}_i$ where $\hat{e}_i$ and $\hat{f}_i$ are basis vectors, then obtain expressions for $|\vec{u}|,|\vec{v}|$ and $\vec{u}.\vec{v}$.

7. Find $L\left(e^{at}\sin bt\right)=?$

8. A and B are real non-zero 3×3 matrices and satisfy the equation $(AB)^T+(BA)^{-1}=0$, then prove that if B is orthogonal then A is antisymmetric.

Group - B

Answer any ***four*** questions out of the following ***six***. 5×4=20

9. State and prove the convolution theorem for Laplace transform.

10. Using Gram-Schmidt orthogonalization process, construct the set of orthonormal vectors of the given set of vectors

$$\vec{X}_1=\begin{pmatrix}0\\0\\1\\1\end{pmatrix};\ \vec{X}_2=\begin{pmatrix}1\\0\\-1\\1\end{pmatrix};\ \vec{X}_3=\begin{pmatrix}1\\2\\0\\2\end{pmatrix};\ \vec{X}_4=\begin{pmatrix}2\\1\\1\\1\end{pmatrix}$$

11. Define the inner product space. Prove the Schwarz inequality

$$|\langle u|v\rangle|\le|u||v|$$

where $|u\rangle$ and $|v\rangle$ are the vectors of a linear vector space in which $\langle u|u\rangle\ge 0$ for all $|u\rangle$ and $\langle v|v\rangle\ge 0$ for all $|v\rangle$.

12. Express the following function in terms of unit step function, $f(t)=\begin{cases}t-1, & 1<t<2\\ 3-t, & 2<t<3\end{cases}$ and find its Laplace transform.

13. Let A be a Hermitian matrix with eigenvalue λ. Then show that λ will be real. Hence also show that the eigenvectors corresponding to the different eigenvalues are orthogonal to each other.

14. Using Laplace transform, find the solution of initial value problem

$$y''+9y=9\theta(t-3),\ y(0)=y'(0)=0$$

where $\theta(t-3)$ is the unit step function.

P.T.O.

Group - C

Answer any ***one*** questions out of the following ***two***. 10×1=10

15. (a) Show that $L\left[\frac{f(t)}{t}\right]=\int_s^\infty \bar{f}(u)\,du$.

(b) Let $\vec{x}$ is a vector in old basis $\hat{e}_i,(i=1,2,...n)$ may be transformed as $\vec{x}=S\vec{x}'$ where $\vec{x}'$ is the vector in transformed basis say $\hat{e}_i'$ and S is the transformation matrix. Consider A is a linear operator represented as matrix in old basis system can be transformed A' in new basis system. Then show that $A'=S^{-1}AS$. Also show A' is diagonal and diagonal elements are the eigenvalues of matrix A. 5+5=10

16. (a) Define a metric tensor. Determine the components of metric tensor in cylindrical coordinates.

(b) If $\vec{v}=\vec{w}\times\vec{r}$. Using tensor index notation show that $\vec{\nabla}\times\vec{v}=\vec{\nabla}\times(\vec{w}\times\vec{r})=2\vec{w}$ where $\vec{w}$ is a constant vector. (2+4)+4=10

OR

[Applied Dynamics]

Full Marks : 40 Time : Two Hours

Group - A

Answer any ***five*** questions : 2×5=10

1. What is a fixed point in a phase portrait? What does it signify?
2. Discuss the stability of the fixed points for : $\dot{x}=\sin x$.
3. Plot the phase portrait for the equation : —

$$\frac{dN}{dt}=rN\left(-N^2+aN-b\right).$$

4. What is butterfly effect?
5. Define fractal dimension.
6. What do you mean by autocorrelation?
7. Define the concept of fluid parcel.
8. What is the geometrical structure of strange attractor?

Group - B

Answer any ***four*** questions : 5×4=20

9. (a) Discuss what is chaos?

(b) Show that cantor set is uncountable. 3+2

P.T.O.

10. (a) Show that the Sierpinski carpet has zero area.

(b) Find the box dimension of von Koch snowflake. 2+3

11. (a) Compare viscous and inviscid flow.

(b) Determine the flow dimensionality of fluid flow through (i) a circular pipe and (ii) a diverging duct. 2+3

12. Find the streamlines for the velocity field $\vec{v} = (-ky, kx, 0)$, where k is a constant. Suppose you are sucking water through a straw. Discuss the forces acting on a fluid element just below the open end of the straw. 3+2

13. Divide the closed unit interval [0, 1] into four quarters. Delete the open second quarter from the left. This produces a set S_1. Repeat this construction indefinitely, i.e. generate S_{n+1} from S_n by deleting the second quarter of each of the intervals in S_n. Sketch the sets S_1, S_2, S_3 and S_4. Compute the box dimension of the limiting set S_∞. Is the limiting set self-similar? 2+2+1

14. Establish the continuity equation for incompressible fluid flow. 5

Group - C

Answer any ***one*** question : 10×1=10

15. Use linear stability analysis to classify the fixed points of the following systems : 2½×4

(a) $\dot{x} = \tan x$

(b) $\dot{x} = 1 - e^{-x^2}$

(c) $\dot{x} = \ln x$

(d) $\dot{x} = ax - x^3$

16. For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and stability. 2½×4

(a) $\dot{x} = 2 + \sin x$

(b) $\dot{x} = -\sinh x$

(c) $\dot{x} = r + x - x^3$, for various values of "r"

(d) $\dot{x} = x(1-x)$

P.T.O.

OR

[Atmospheric Physics]

Full Marks : 40 Time : Two Hours

Group - A

Answer any ***five*** of the following : 2×5=10

1. What are sea breeze and land breeze?
2. State the Bouguer-Lambert law.
3. Distinguish between Mie scattering and Rayleigh scattering.
4. Write the applications of LiDAR.
5. What is mesoscale circulation?
6. Explain Lamb wave.
7. What are cyclones and anticyclones?
8. Write the principle of radiometry.

Group - B

Answer any ***four*** questions : 5×4=20

9. Explain absolute vorticity and potential vorticity.
10. Write down the radar equation. Explain briefly the mechanism of signal processing and detection in radar system.

11. Give a brief note on the acoustic wave and buoyancy wave.
12. Classify and write down the properties of aerosols.
13. What are the compositions of the atmosphere? Write down the different instruments required for meteorological observation. What is the effective temperature of Earth?
14. Describe the propagation of atmospheric gravity waves in a non-homogeneous medium.

Group - C

Answer any ***one*** question : 10×1=10

15. Describe the thermal structure of the earth's atmosphere. Explain in detail the Greenhouse effect. 5+5
16. Write the vectorial form of the momentum equation in rotating co-ordinate system. What are atmospheric oscillations? 6+4

P.T.O.

OR

[Classical Dynamics]

Full Marks : 60 Time : Three Hours

Group - A

Answer any ***ten*** questions : 2×10=20

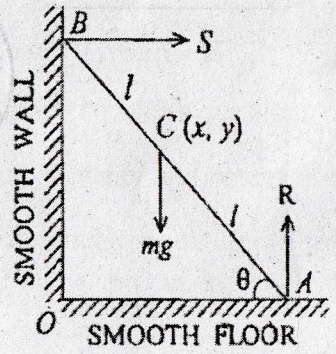
1. What is the difference between coordinates and generalized coordinates?

2. Determine the number of degrees of freedom for :

   (a) 4 particles moving freely in space

   (b) 4 particles moving freely in a plane

3. A ladder of length $2l$ slides down a smooth wall along a smooth floor. Write down its Lagrangian.



4. Write down the Lagrangian of a particle moving under central force.

5. Plot the variation of effective potential with distance in case of bounded motion of a particle under central force.

6. Find the Hamiltonian corresponding to Lagrangian

   $L=\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_1\dot{q}_2+\dot{q}_2^{\,2}\right)-V(q)$; q, q_1, q_2 are generalized coordinate.

7. What is the dimension of generalized velocity, if generalized coordinate has dimension of momentum? Justify it.

8. Write down the criteria for Hamiltonian being equal to the total energy of the system.

9. The length of a spaceship is measured to be exactly half its proper length. What is the speed of the spaceship relative to the observer on earth?

10. Two lumps of clay, each of rest mass m, collide head-on with velocity 3c/5. They stick together and come to rest. What is the mass of the composite lump?

P.T.O.

11. A body of mass m_0 at rest breaks up spontaneously into two parts, having rest masses m_1 and m_2 and speed v_1 and v_2 respectively. Show that $m_0 > (m_1 + m_2)$.

12. When kinetic energy and potential energy are expressed in normal coordinates, cross terms of normal coordinates are present — True or False?

13. State the conditions required to define stable equilibrium.

14. Is Navier-Stokes the continuity equation? Justify your answer.

15. Give the idea about Doppler effect from four-vector perspective.

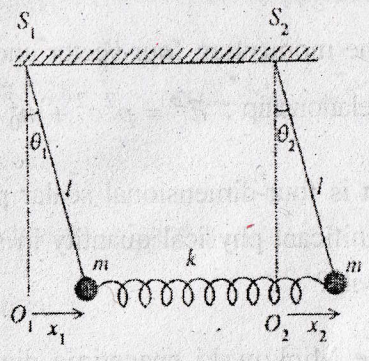
Group - B

Answer any ***four*** questions : 5×4=20

16. Show that for a charged particle under electromagnetic force, the potential is velocity-dependent, and find the generalized momenta for the system. 4+1

17. Show that if Lagrangian of a system does not involve time explicitly, then Hamiltonian is conserved. Does this imply energy is conserved? Explain. 3+2

18. Find the normal frequencies for the coupled pendulum as shown in the figure. 5



19. Find the value of effective potential of an object moving in a circular orbit under the action of central force defined as $F = \frac{-k}{r^2}$, where k is constant and r is the distance. 5

20. Water flows through a narrow horizontal tube, length 50 cm under a pressure 5000 dyne/cm^2. If the radius of the tube = 0.1 cm, density of water is 1g/cc and rate of flow is 1000 cc/sec, check whether the liquid has streamline or turbulent flow of motion. 5

21. Derive the expression for continuity of liquid flow. 5

P.T.O.

Group - C

Answer any ***two*** questions : 10×2=20

22. (a) Define momentum four-vector and hence establish the relationship : $E^2 = p^2c^2 + m_0^2 c^4$. 2+2

(b) What is four-dimensional scalar product? Why is it significant physical quantity in special theory of relativity? 2+1

(c) In the Minkowski spacetime diagram, draw the world line of a particle which first moves with constant velocity $c/4$ along X-axis for 5 seconds and then suddenly comes to rest. 3

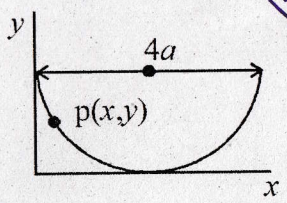
23. Describe the motion of a particle of mass m constrained to move on the surface of a cylinder of radius "a" and attracted towards the origin by a force which is proportional to the distance of the particle from the origin. Show that motion of the particle in z-direction is simple harmonic. Find the corresponding time period T. 5+3+2

24. A bead slides on a wire under th effect of gravity in the shape of a cycloid described by equations

$$x = a(\theta - \sin\theta)$$

$$y = a(1 + \cos\theta),$$

where $0 \le \theta \le 2\pi$



Find (a) Lagrangian, and

(b) Equation of motion.

Neglect friction. 4+6

25. Find all the normal frequency/frequencies in a linear triatomic symmetric molecule (CO_2). Draw the schematic representation of vibration and justify it. 5+2+3
