B.Sc./5th Sem (H)/PHS/23(CBCS)

2023

5th Semester Examination

PHYSICS (Honours)

Paper : C 11-T

[Quantum Mechanics and Applications]

[CBCS]

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Attempt any ***five*** questions : 2×5=10

1. At time $t = 0$, a particle is represented by the wave function :

$$\begin{aligned} \varphi(x,0) &= A(x/a), 0 \le x \le a \\ &= A(b-x)/(b-a), a \le x \le b \\ &= 0, \text{otherwise} \end{aligned}$$

$\sqrt{\frac{3}{b}}$

Find the value of A. 2

P.T.O.

2. Can the following function $f(x)$ be considered as a wave function?

$[f(x)]^2 = 4x \exp(-x)$

Explain. 2

3. Show that the operator (d/dx) can have only imaginary eigenvalues. 2

4. Suppose wave function of a particle is given by $\varphi(x) = Ae^{\frac{ip_0 x}{\hbar}}$. Find out the corresponding wave function in momentum space. 2

5. What is zero-point energy? Why can't it be zero? 2

6. Consider three bosons inside an infinite 1-D potential well of dimension "a". Write down the wave function for the ground state and 1st excited state. 1+1

7. What was the conclusion of Stern-Gerlach experiment? 2

8. The state space of a system is described by the orthonormal basis vectors $|e_1\rangle$ and $|e_2\rangle$. Consider the operator : $A = k(|e_1\rangle\langle e_1| - i|e_1\rangle\langle e_2| + i|e_2\rangle\langle e_1|) - |e_2\rangle\langle e_2|)$, where k is a real constant. Find the eigenvalues of A. 2

Group - B

Attempt any ***four*** questions : 5×4=20

9. What do you mean by spin-orbit coupling? Find out the Lande g-factor for $^3P_2$. What is the main difference

between orbital angular momentum and spin angular momentum? 2+2+1

10. Consider a system whose wave function is given by :

$$\Psi(x,0) = \frac{5}{\sqrt{50}}\phi_0(x) + \frac{4}{\sqrt{50}}\phi_1(x) + \frac{3}{\sqrt{50}}\phi_2(x)$$

where $\phi_n(x)$'s are the eighenfunction of Hamiltonian for a harmonic oscillator. Find the average energy of the system and expected number of energy quanta present in the system. 4+1

11. Find the eigenvalues and eigenstates of the component of the spin operator $\vec{S}$ of an electron along the direction of unit vector $\hat{n}$, assume $\hat{n}$ lies in the XZ-plane. 5

12. Prove that, $\hat{L}_{\pm}|l,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm1)}\,|l,m\pm1\rangle$
Using this relation, find out expectation value of L_+ for the state 3+2

$$\frac{1}{\sqrt{5}}Y_{20} + \sqrt{\frac{2}{5}}\left(Y_{2,-1} - Y_{2,1}\right)$$

13. Explain how Sodium D_1 line $(3\,^2P_{1/2} \rightarrow 3\,^2S_{1/2})$ and D_2 line $(3\,^2P_{3/2} \rightarrow 3\,^2S_{1/2})$ split into four and six components respectively under anomalous Zeeman effect. 5

P.T.O.

14. (a) Consider the wave function $\Psi(x) = Axe^{-\frac{m\omega x^2}{2\hbar}}$, corresponding to the potential $V(x) = \frac{m\omega^2 x^2}{2}$.

(i) Find the normalization constant.

(ii) Find the probability of finding the particle in the classically forbidden region in this state.

[Hint : $\frac{2}{\sqrt{\pi}}\int_0^1 u^2 e^{-u^2} du = 0.214$]

(b) Find the term symbol of Nitrogen Atom.

1+2+2

Group - C

Attempt any ***one*** question : 10×1=10

15. Consider a particle of mass m moving in a one-dimensional harmonic oscillator potential:

(a) Define $\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}}[m\omega\hat{x} + i\hat{p}]$ and

$\hat{a}^{\dagger} = \frac{1}{\sqrt{2m\hbar\omega}}[m\omega\hat{x} - i\hat{p}]$.

Find the commutation relation between a and $a^{\dagger}$. 2

(b) Express Hamiltonian of LHO in terms of above two operators. 2

(c) Prove that,

$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$, $\hat{a}^{\dagger}|n\rangle = \sqrt{(n+1)}|n+1\rangle$ 3

(d) Calculate the uncertainty product for position and momentum operator in the 5th excited state, i.e. n = 5. Is it larger or smaller than that for the ground state? Comment. 2+1

16. Consider a hydrogen atom which is in its ground state; the ground state wave function is given by :

$\Psi(r,\theta,\varphi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$, where a_0 is the Bohr radius.

(a) Find out the most probable distance between the electron and proton. 3

(b) Find out the average distance between the electron and proton. 3

(c) How many degenerate states are there for n = 3 state of hydrogen atom? Write down all the degenerate states in $|n, l, m\rangle$ notation. 1+3
