

2023

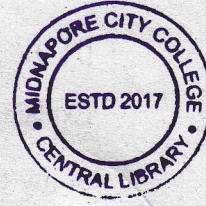
5th Semester Examination

PHYSICS (Honours)

Paper : C 11-T

[Quantum Mechanics and Applications]

[CBCS]



Full Marks : 40

Time : Two Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

**Group - A**Attempt any *five* questions :  $2 \times 5 = 10$ 

1. At time  $t = 0$ , a particle is represented by the wave function :

$$\begin{aligned}\varphi(x, 0) &= A(x/a), 0 \leq x \leq a \\ &= A(b-x)/(b-a), a \leq x \leq b \\ &= 0, \text{ otherwise}\end{aligned}$$

Find the value of  $A$ .

2

 $\sqrt{\frac{3}{b}}$ 

P.T.O.



( 2 )

2. Can the following function  $f(x)$  be considered as a wave function?

$$[f(x)]^2 = 4x \exp(-x)$$

Explain.

2

3. Show that the operator  $(d/dx)$  can have only imaginary eigenvalues.

2

4. Suppose wave function of a particle is given by

$$\psi(x) = Ae^{i\frac{p_0 x}{\hbar}}$$

Find out the corresponding wave function in momentum space.

2

5. What is zero-point energy? Why can't it be zero?

2

6. Consider three bosons inside an infinite 1-D potential well of dimension "a". Write down the wave function for the ground state and 1<sup>st</sup> excited state.

1+1

7. What was the conclusion of Stern-Gerlach experiment?

2

8. The state space of a system is described by the orthonormal basis vectors  $|e_1\rangle$  and  $|e_2\rangle$ . Consider the operator :  $A = k(|e_1\rangle\langle e_1| - i|e_1\rangle\langle e_2| + i|e_2\rangle\langle e_1|) - |e_2\rangle\langle e_2|$ , where  $k$  is a real constant. Find the eigenvalues of  $A$ .

2

**Group - B**

Attempt any **four** questions :

5×4=20

9. What do you mean by spin-orbit coupling? Find out the Lande g-factor for  $^3P_2$ . What is the main difference

( 3 )

between orbital angular momentum and spin angular momentum?  
2+2+1

10. Consider a system whose wave function is given by :

$$\Psi(x, 0) = \frac{5}{\sqrt{50}}\phi_0(x) + \frac{4}{\sqrt{50}}\phi_1(x) + \frac{3}{\sqrt{50}}\phi_2(x)$$

where  $\phi_n(x)$ 's are the eigenfunction of Hamiltonian for a harmonic oscillator. Find the average energy of the system and expected number of energy quanta present in the system.  
4+1

11. Find the eigenvalues and eigenstates of the component of the spin operator  $\bar{S}$  of an electron along the direction of unit vector  $\hat{n}$ , assume  $\hat{n}$  lies in the XZ-plane. 5

12. Prove that,  $\hat{L}_x |l, m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)} |l, m\pm 1\rangle$   
Using this relation, find out expectation value of  $L_x$  for the state  
3+2

$$\frac{1}{\sqrt{5}}Y_{20} + \sqrt{\frac{2}{5}}(Y_{2,-1} - Y_{2,1})$$



13. Explain how Sodium  $D_1$  line ( $3^2P_{1/2} \rightarrow 3^2S_{1/2}$ ) and  $D_2$  line ( $3^2P_{3/2} \rightarrow 3^2S_{1/2}$ ) split into four and six components respectively under anomalous Zeeman effect. 5

P.T.O.

Handwritten notes and calculations at the bottom of the page, including a diagram of a Zeeman splitting pattern with levels labeled  $3/2, 1/2, 3/2, 1/2$  and  $3/2, 1/2$ .



( 4 )

14. (a) Consider the wave function  $\Psi(x) = Axe^{\frac{m\omega^2 x^2}{2\hbar}}$ , corresponding to the potential  $V(x) = \frac{m\omega^2 x^2}{2}$ .

(i) Find the normalization constant.

(ii) Find the probability of finding the particle in the classically forbidden region in this state.

[Hint :  $\frac{2}{\sqrt{\pi}} \int_0^1 u^2 e^{-u^2} du = 0.214$ ]

- (b) Find the term symbol of Nitrogen Atom.

1+2+2

**Group - C**

Attempt any *one* question :

10×1=10

15. Consider a particle of mass  $m$  moving in a one-dimensional harmonic oscillator potential.

(a) Define  $\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}} [m\omega\hat{x} + i\hat{p}]$  and

$$\hat{a}^\dagger = \frac{1}{\sqrt{2m\hbar\omega}} [m\omega\hat{x} - i\hat{p}].$$

Find the commutation relation between  $a$  and  $a^\dagger$ .

2

- (b) Express Hamiltonian of LHO in terms of above two operators.

2

( 5 )

(c) Prove that,

$$\hat{a}|n\rangle \geq \sqrt{n}|n-1\rangle > \hat{a}^\dagger|n\rangle \geq \sqrt{(n+1)}|n+1\rangle >$$

3

(d) Calculate the uncertainty product for position and momentum operator in the 5<sup>th</sup> excited state, i.e.  $n = 5$ . Is it larger or smaller than that for the ground state? Comment.

2+1

16. Consider a hydrogen atom which is in its ground state; the ground state wave function is given by :

$$\Psi(r, \theta, \phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \text{ where } a_0 \text{ is the Bohr radius.}$$

(a) Find out the most probable distance between the electron and proton.

3

(b) Find out the average distance between the electron and proton.

3

(c) How many degenerate states are there for  $n = 3$  state of hydrogen atom? Write down all the degenerate states in  $|n, l, m\rangle$  notation.

1+3