

বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examination 2023

(Under CBCS Pattern)

Semester — II

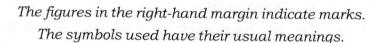
Subject: MATHEMATICS

Paper: GE-2T

(Algebra)

Full Marks: 60

Time: 3 hours



Answer from all the Groups as directed.

GROUP-A

- 1. Answer any ten questions from the following: $2 \times 10 = 20$
 - (a) State and prove the triangle inequality of complex numbers.

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(Turn Over)





(2)

- (b) Find the product of all the values of $(1+i)^{\frac{4}{5}}$.
- (c) Apply Descartes' rule of signs to find the nature of the roots $x^7 + x^5 x^3 = 0$.
- (d) If a, b, x and y all are positive real numbers, then prove that

$$\frac{ax+by}{a+b} \ge \frac{(a+b)xy}{ay+bx}$$

(e) Find the number of multiple roots of the equation

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

- Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 3 & 1 & 2 & 3 \\ 1 & 7 & 2 & 1 \end{pmatrix}$.
- (g) Is the union of two subspaces of a vector space V form a subspace of V? Justify.
- (h) Find the remainder when $1!+2!+3!+\cdots+80!$ is divided by 30.
- (i) Let $W = \{(x, y, z) \in \mathbb{R}^3 \mid x = 0 \text{ or } y = 0\}$. Is W a subspace of \mathbb{R}^3 ? Why?
- (j) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by T(x,y) = (2x-y, -8x+4y). Does (5,10) belong to $\ker(T)$? Justify.
- (k) Using the principle of induction, prove that $(3+\sqrt{5})^n+(3-\sqrt{5})^n$ is an even integer for all $n \in \mathbb{N}$.

(a

- (1) Let α , β , γ , δ be the roots of the equation $x^4 + qx^2 + rx + s = 0$. Find $\sum_{\alpha} \frac{1}{\alpha^2}$.

$$\begin{cases} x + y + z = kx \\ x + y + z = ky \\ x + y + z = kz \end{cases}$$



will have non-trivial solution.

- (n) Let A be a 7×6 matrix such that Ax = 0 has only the trivial solution.
- (o) Show that the map $T: \mathbb{R}^3 \to \mathbb{R}$ defined by $T(x,y,z) = x+y+z, \ \forall x,y,z \in \mathbb{R}$ is a linear map.

GROUP-B

- **2.** Answer *any* **four** questions from the following: $5\times4=20$
- (a) Solve the equation by Ferrari's method $x^4 2x^2 + 8x 3 = 0$.
- (b) If a, b, c are positive rational numbers, then show that

$$a^ab^bc^c \geq \left(\frac{a+b}{2}\right)^{\frac{a+b}{2}}\left(\frac{b+c}{2}\right)^{\frac{b+c}{2}}\left(\frac{c+a}{2}\right)^{\frac{c+a}{2}} \geq \left(\frac{a+b+c}{3}\right)^{a+b+c}$$

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If a, b, c are the roots of the equation roots are $(a-b)^2$, $(b-c)^2$, $(a-c)^2$. $x^3 + qx + r = 0$, then find the equation whose

- (d) If a, b, c are positive real numbers such that a+b+c=1, then show that $\sqrt{4a+1}+\sqrt{4b+1}+\sqrt{4c+1}<5$.
- (e) State Cayley-Hamilton theorem for matrix.

Using it, compute the inverse of | 1 1+4=5

(f)Find a basis and dimension of the subspace $S \cap T$ of \mathbb{R}^4 where $T = \{(x, y, z, w) \in \mathbb{R}^4 | 2x + y - z + w = 0\}$ $S = \{(x, y, z, w) \in \mathbb{R}^4 | x + y + z + w = 0\}$ and

GROUP-C

(c)

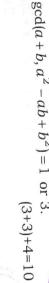
- **3.** Answer *any* **two** questions from the following: 10×2=20
- (a) (i) Let $f(x) = x^4 + 6x^2 + 14x^2 + 22x + 5$. Find expressed the equation f(x) = 0 α , β and λ so that f(x) may be $(x^2+3x+\lambda)^2-(\alpha x+\beta)^2$. Hence solve in the form

(Continued)

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(ii) If gcd(a,b) = 1, then show that $gcd(a+b, a^2-ab+b^2)=1$ or 3.



(i) Define orthogonal matrix. Prove that if \(\lambda \) be an eigenvalue of a real orthogonal matrix A, then $\frac{1}{\lambda}$ is also an eigenvalue

(b)

(ii) Let A be a 3×3 real matrix with its

eigenvectors
$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$
, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ corresponding to eigenvalues 2, 3 and 1 respectively. Find A. $(1+4)+5=10$

(i) Let $(\alpha_1,\alpha_2,\alpha_3)$ and $(\beta_1,\beta_2,\beta_3)$ be the order bases of the real vector space Vand W respectively. A linear mapping Find the matrix of T relative to the order $T(\alpha_1) = \beta_1 + \beta_2, \ T(\alpha_2) = \beta_2 + \beta_3, \ T(\alpha_3) = \beta_3.$ $T:V\to W$ maps the basis vectors as chosen order bases. matrix of T^{-1} relative to the same W. Deduce that T is invertible. Find the bases $(\alpha_1, \alpha_2, \alpha_3)$ of V and $(\beta_1, \beta_2, \beta_3)$ of

(Turn Over)



(ii) Find the dimension of the subspace S of \mathbb{R}^3 defined by

$$S = \{(x, y, z) \in \mathbb{R}^3 : x + 2y = z, \\ 2x - y + 3z = 0\}$$

6+4=10

(d) (i) Let R_1 be a relation defined on the set of integers $\mathbb Z$ such that

$$R_1 = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} : x-y = 5n\}$$
 Show that R_1 is an equivalence relation.

- (ii) Find the roots of the equation $z^n = (z+1)^n$, where z is a complex number and n is a positive integer.
- (iii) Let V be a vector space of functions from \mathbb{R} to \mathbb{R} . Verify whether the transformation $T:V\to\mathbb{R}^2$ given by $T(f)=(f(0),\,f(1)+1)$ is linear.

4+3+3=10



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BL23(034)-1290