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B.Sc./6th Sem (H)/MATH/23(CBCS)

2023

6th Semester Examination

MATHEMATICS (Honours)

Paper : DSE 4-T

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[Mathematical Modelling]

Group - A

Answer any ***ten*** questions : 2×10=20

1. What is Monte Carlo simulation?

2. Use the middle-square method to generate 2 random numbers considering the seed $x_0 = 3043$.

3. Find $L^{-1}\left\{\dfrac{s+2}{s^2(s+3)}\right\}$.

4. Show that Laplace transform of the function $f(t) = t^n$, $-1 < n < 0$, exists, but it is not piecewise continuous on every finite subinterval in the range $t \geq 0$.

P.T.O.

5. The cost of any non-basic variable can be reduced without limit, without affecting the optimal basic feasible solution to the LPP. Justify.

6. If $L\{f(t)\}=\dfrac{50s+3}{s^4+3s^2+(k-4)s}$ and $\lim_{t\to\infty} f(t)=1$ then find the value of k where k is a constant.

7. Let $f(t)$ be the continuous function on $[0,\infty]$ whose Laplace Transform exists. If $f(t)$ satisfies $\int_0^t (1-\cos(t-u))f(u)\,du=t^4$ then find $f(t)$.

8. Is the solution $\left(1, \frac{1}{2}, 0, 0, 0\right)$ a basic solution of the equations

$$x_1+2x_2+x_3+x_4=2$$

$$x_1+2x_2+\frac{1}{2}x_3+x_5=2\,?$$

9. What is meant by singularity of a linear ordinary differential equation?

10. Write the general expression of $P_n(z)$.

11. What do you mean cycling in linear congruence?

12. Prove that $L^{-1}\left\{\dfrac{f(s)}{s^2}\right\}=\int_0^t\int_0^v F(u)\,du\,dv$.

13. Does the Laplace transform of $\dfrac{\cos at}{t}$ exist?

14. What is a probabilistic process?

15. Why we generate random number?

Group - B

Answer any ***four*** questions : 5×4=20

16. How do you generate random numbers between 0 and 1 that follow uniform distribution using linear congruence method?

17. Using Monte Carlo simulation, write an algorithm to calculate that part of the volume of an ellipsoid

$$\frac{x^2}{2}+\frac{y^2}{4}+\frac{z^2}{8}\le 16$$

18. In the LPP

Maximize $Z=3x_1+5x_2$

Subject to $x_1+x_2\le 1$

$2x_1+3x_2\le 1$

$x_1, x_2\ge 0,$

obtain the variation of $c_j\,(j=1,2)$ without changing the optimality of solution.

19. Find $L^{-1}\left\{\frac{1}{\sqrt{p}(p-a)}\right\}$ by the convolution integral.

20. Generate 15 random numbers using middle-square method taking $x_0 = 3043$.

21. Prove the Final value theorem $\underset{t\to\infty}{Lt} f(t) = \underset{s\to 0}{Lt}\, sF(s)$.

Group - C

Answer any ***two*** questions : 10×2=20

22. (i) Use Laplace transform to solve the following initial-value problem

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = h(t),\ y(0) = 0,\ y'(0) = 0 \text{ where}$$

$$h(t) = \begin{cases} 2, & 0 < t < 4 \\ 0, & t > 4 \end{cases}$$

(ii) Prove that $L\{\sinh at \cos at\} = \frac{a(s^2 - 2a^2)}{s^4 + 4a^4}$. 7+3

23. (i) Write a Monte Carlo simulation algorithm for a harbour with unloading facilities for ships finding the answers of the following questions :

1. What is the average and maximum times per ship in the harbour?

2. What are the average and maximum waiting times per ship?
3. What percentage of the time are the unloading facilities idle?

(ii) What are the disadvantages of linear congruence method to generate random numbers? 7+3

24. (i) Find the Laplace inverse of the function $\frac{1}{(p+a)^3}$.

(ii) Solve the following LPP using simplex method

Maximize $Z = 6x + 4y$

subject to $-x + y \le 12$

$x + y \le 24$

$2x + 5y \le 80$

$x, y \ge 0.$ 3+7

25. Find the solution of the Bessel differential equation of order λ at the neighbourhood of $x = 0$. Discuss the case when $\lambda = 0$. 8+2

P.T.O.

OR

[Differential Geometry]

Group - A

Answer any ***ten*** questions : 2×10=20

1. State Euler's theorem on a surface.
2. Define a space curve. Give an example of a space curve and describe its motion.
3. What is torsion? How is torsion related to the curvature of a space curve?
4. What is Rodrigues' formula in differential geometry?
5. Give an example of a surface on which every point is umbilic.
6. What is the relation of curvature and torsion of a helix?
7. Write down the relation between the three fundamental forms of the surface, immersed in E^3.
8. Prove that any part of a straight line on surface is a geodesic.
9. Define the torsion of a geodesic, and explain its significance.
10. Define a minimal surface, and give an example of a minimal surface.
11. Define a developable surface, and give an example of a developable surface.

12. Give an example of a surface of revolution that has infinitely many geodesics.
13. Define the Jacobi equation for geodesics, and explain how it relates to the variation of geodesics.
14. Define the tangent indicatrix of a space curve, and explain its relationship to the developable associated with the curve.
15. Define a ruled surface, and give an example of a ruled surface.

Group - B

Answer any ***four*** questions : 5×4=20

16. Compute the curvature and torsion of the circular helix $\gamma(\theta)=(a\cos\theta, a\sin\theta, b\theta)$, where $-\infty<\theta<\infty$ and a, b are constants.
17. State the Gauss-Bonnet theorem, and explain its significance in differential geometry.
18. Give an example of a surface that has constant mean curvature, and explain its geometric properties.
19. Prove that any tangent plane to the surface $a\left(x^2+y^2\right)+xyz=0$ meets it again in a conic whose projection on the $xy-$ plane is a rectangular hyperbola.

P.T.O.

20. Define the Weingarten map of a surface, and explain how it relates to the first and second fundamental forms.

21. Calculate the fundamental magnitudes for the surface of revolution $x = u\cos v$, $y = u\sin v$, $z = f(u)$ with u, v are parameters.

Group - C

Answer any *two* questions : $10\times2=20$

22. (i) Prove that the curves of the family $\frac{v^3}{u^2} = \text{constant}$ are geodesics on a surface metric $v^2du^2 - 2uvdudv + 2u^2dv^2$; $(u > 0,\ v > 0)$.

(ii) Determine the principal curvatures of the right circular cylinder. 5+5

23. (i) A geodesic on the ellipsoid of revolution $\frac{x^2+y^2}{a^2} + \frac{z^2}{c^2} = 1$ crosses a meridian at an angle θ at a distance u from the axis. Prove that at the point of crossing it makes an angle $\cos^{-1}\frac{cu\cos\theta}{\left[a^4 - u^2\sqrt{a^2-c^2}\right]}$ with the axis.

(ii) Define a smooth surface. Prove that surface of a sphere is a smooth surface. 5+(1+4)

24. (i) Prove that any curve is a geodesic on the surface generated by its binormals and an asymptotic line on the surface generated by its principal normals.

(ii) For the curve $x = a(3u - u^3)$, $y = 3au^2$, $z = a(3u + u^3)$ show that the Curvature and torsion are equal. 5+5

25. (i) Define the mean curvature of a surface, and explain its relationship to the principal curvatures.

(ii) Define the asymptotic curvature of a curve on a surface, and explain its relationship to the principal curvatures. 4+6

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OR

[Bio Mathematics]

The symbols have their usual meaning.

Illustrate the answers wherever necessary.

Group - A

Answer any ***ten*** of the following : 2×10=20

1. Discuss the Malthus population growth model for single species.
2. Define the stability conditions at an equilibrium point of a single species growth model.
3. What do you mean by Allee effect?
4. Define proportional harvesting.
5. What do mean by functional response and numerical response of a prey-predator system?
6. Write down Holling type-I and II response functions with their graphical representations.
7. Define Horizontal and Vertical transmissions.
8. Discuss the existence criteria of unique equilibrium point for a first order linear non homogeneous difference equation.

9. Define the stability and asymptotic stability for an equilibrium point.
10. Consider the difference equation $x_{t+1}+x_t^3+x_t=0$. Is $x=0$ stable?
11. Define periodic orbit and limit cycle.
12. Does the system $x''+\left(x^2+x'^2-\lambda\right)x'+x=0$, $\lambda\in R$ exhibit a Holf bifurcation? Justify.
13. Find the steady states of the following systems of equations :

$$\frac{dx}{dt}=x^2-y^2;\ \frac{dy}{dt}=x(1-y).$$

14. Write down the basic assumptions for Lotka-Volterra prey-predator model.
15. Define Routh-Hurwitz Criteria for stability of k species system.

Group - B

Answer any ***four*** of the following : 5×4=20

16. Propose a mathematical model describing the interaction between microbes and nutrient. Determine the stability of its equilibrium.

P.T.O.

17. Consider the model :

$$\frac{dN}{dt} = rN(K-N)(N-M)$$

where $r > 0$ and $0 < M < K$. (i) Express the intrinsic growth rate $g(N)$ as a polynomial in N and find the co-efficients a_1, a_2, a_3. (ii) Show that $N = 0$, $N = K$, and $N = M$ are steady states and determine their stability.

18. Show it is possible, by introducing dimensionless variables, to rewrite the Lotka-Volterra equation as

$$\frac{dv}{dt} = v(1-e); \frac{de}{dt} = \alpha e(v-1)$$

where v = victims and e = exploiters.

19. Show that in an SIR model with carriers who show no symptoms of the disease, the disease always remains endemic.

20. The population of a certain species subjected to a specific kind of predation is modeled by the difference equation $u_{t+1} = a\frac{u_t^2}{b^2 + u_t^2}$, $a > 0$.

Determine the equilibria and show that if $a^2 > 4b^2$ it is possible for the population to be driven to extinction if it becomes less than a critical size which you should find.

21. A system consists of two substances. The activator enhances its own synthesis as well as that of the inhibitor. The inhibitor causes the formation of both substances to decline. Several versions have been studies, among them is the following system :

$$\frac{dx}{dt} = \rho + \frac{x^2}{y} - x; \frac{dy}{dt} = x^2 - \gamma y.$$

(i) Find the steady state for this system.

(ii) Show that if the input of activator is sufficiently small compared to the decay rate of inhibitor, the system is an activator-inhibitor system.

Group - C

Answer any *two* of the following : 10×2=20

22. Describe Spruce budworm model with appropriate assumptions. Nondimensionalize your model with appropriate transformation. Describe the existence of equilibrium points and their qualitative behaviour graphically.

23. Holling-Tanner type predator prey model is considered in the form

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - \frac{exy}{m+x}; \frac{dy}{dt} = \theta y\left(1 - \frac{gy}{x}\right).$$

Describe the underlying assumption of this model. Find the interior equilibrium and give its stability conditions.

P.T.O.

24. Write down a diffusion model for two interacting populations. Determine the necessary and sufficient condition for diffusive instability.

25. With proper assumptions, propose a SIRS model with horizontal and perfect vertical transmission. Determine the equilibrium points of the proposed model and write the stability conditions of the endemic equilibrium point.
