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B.Sc./4th Sem (H)/MATH/23(CBCS)

2023

4th Semester Examination

MATHEMATICS (Honours)

Paper : C 9-T

[Multivariate Calculus]

[CBCS]

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any *ten* questions from the following : $2\times10=20$

(a) Evaluate $\int_1^2 \vec{r}\times\frac{d^2\vec{r}}{dt^2}dt$, given that $\vec{r}=t\hat{i}+t^2\hat{j}+t^3\hat{k}$.

(b) Find the total work done in moving a particle in a force field given by $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$ along the curve $x=t^2+1$, $y=2t^2$, $z=t^3$ from $t=l$ to $t=2$.

(c) If $z=x^3-xy+y^3$, $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$.

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(d) Show that at (0, 0) both the repeated limits exist and are equal for the function $f(x, y) = \frac{xy}{x^2 + y^2}$.

(e) Show that

$$f(x, y) = \begin{cases} xy\frac{x^2 - y^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0).

(f) Find the directional derivative of $f(x, y) = 2x^2 - xy + 5$ at (1, 1) in the direction of the unit vector $\beta = \frac{1}{5}(3, -4)$.

(g) Find $\iint\limits_R y\,dx\,dy$ over the part of the plane bounded by the lines $y = x$ and the parabola $y = 4x - x^2$.

(h) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point (2, −2, 3).

(i) Define an open set $S \subseteq R^n$ and limit point of a set $S \subseteq R^n$.

(j) Express Green's theorem in the plane in vector notation.

(k) Furnish the sufficient condition of differentiability.

(l) Compare the continuity of a function of single variable and function of double variable.

(m) Write down the necessary and sufficient condition of integrability.

(n) What is the area between the curves $y = x^2$ and $x - 1 = y^2$?

(o) State Stokes' theorem and interpret it.

2. Answer any ***four*** questions from the following : 5×4=20

(a) Prove that the function $f(x, y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither a maximum nor minimum at origin.

(b) Determine the constant a so that the vector $\vec{V} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ is solenoidal.

(c) If $f(x, y) = \begin{cases} 0, & x^2 - y^2 = 0 \\ xy\frac{x^2 - y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \end{cases}$, prove that $f_{xy} = f_{yx}$ at (0, 0).

(d) If $z = xf(x + y) + yg(x + y)$, prove that

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0.$$

(e) Find $\iint (x^2+y^2)\sqrt{4a^2-x^2-y^2}$ when the region is the upper half of the circle $x^2+y^2-2ax=0$.

(f) State and prove the Euler's theorem for a homogeneous function of three variables.

3. Answer any *two* questions of the following : 10×2=20

(a) (i) Prove that

$$\iiint \frac{dx\,dy\,dz}{x^2+y^2+(z-2)^2} = \pi\left(2-\frac{3}{2}\log 3\right),$$

extended over the sphere $x^2+y^2+z^2 \le 1$. 6

(ii) Explain the term 'Differentiability' and 'Total Differential' for a function of two variables. 4

(b) (i) Find the constants a and b so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point $(1,-1,2)$. 4

(ii) Verify Stokes' theorem for $\vec{F}=(2y+z)\hat{i}+(x-z)\hat{j}+(y-x)\hat{k}$, over the triangle ABC cut from the plane $x+y+z=1$ by the coordinate planes. 6

(c) (i) The plane $x+y+z=1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the points on

the ellipse that lie closest to and farthest from the origin. 6

(ii) Prove that $\iiint_V \vec{\nabla}\times\vec{B}\,dV = \iint_S \hat{n}\times\vec{B}\,dS$. 4

(d) (i) Using Lagrange multiplier method, prove the inequality $\frac{x+y+z}{3} \ge \sqrt[3]{xyz}$, $x\ge 0$, $y\ge 0$, $z\ge 0$. 6

(ii) Find the length of the arc of the parabola $(y-2)^2=16(x-1)$ measured from the vertex to an extremity of the latus rectum. 4

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