

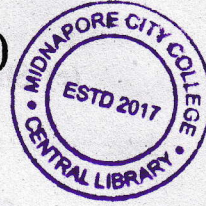
2023

4th Semester Examination
MATHEMATICS (Honours)

Paper : C 9-T

[Multivariate Calculus]

[CBCS]



Full Marks : 60

Time : Three Hours

*The figures in the margin indicate full marks.**Candidates are required to give their answers
in their own words as far as practicable.*1. Answer any *ten* questions from the following :

2×10=20

(a) Evaluate $\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2} dt$, given that $\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$.(b) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.(c) If $z = x^3 - xy + y^3$, $x = r \cos \theta$, $y = r \sin \theta$, find

$$\frac{\partial z}{\partial r} \text{ and } \frac{\partial z}{\partial \theta}.$$

P.T.O.

(d) Show that at $(0, 0)$ both the repeated limits exist and are equal for the function $f(x, y) = \frac{xy}{x^2 + y^2}$.

(e) Show that

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$.

(f) Find the directional derivative of $f(x, y) = 2x^2 - xy + 5$ at $(1, 1)$ in the direction of the unit vector $\beta = \frac{1}{5}(3, -4)$.

(g) Find $\iint_R yx dx dy$ over the part of the plane bounded by the lines $y = x$ and the parabola $y = 4x - x^2$.

(h) Find a unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

(i) Define an open set $S \subseteq R^n$ and limit point of a set $S \subseteq R^n$.

(j) Express Green's theorem in the plane in vector notation.

(k) Furnish the sufficient condition of differentiability.

(l) Compare the continuity of a function of single variable and function of double variable.

(m) Write down the necessary and sufficient condition of integrability.

(n) What is the area between the curves $y = x^2$ and $x - 1 = y^2$?

(o) State Stokes' theorem and interpret it.

2. Answer any four questions from the following : $5 \times 4 = 20$

(a) Prove that the function

$f(x, y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither a maximum nor minimum at origin.

(b) Determine the constant a so that the vector $\vec{V} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ is solenoidal.

(c) If $f(x, y) = \begin{cases} 0, & x^2 - y^2 = 0 \\ xy \frac{x^2 - y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \end{cases}$, prove

that $f_{xy} = f_{yx}$ at $(0, 0)$.

(d) If $z = xf(x + y) + yg(x + y)$, prove that

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0.$$

P.T.O.

(e) Find $\iiint (x^2 + y^2) \sqrt{4a^2 - x^2 - y^2}$ when the region

is the upper half of the circle $x^2 + y^2 - 2ax = 0$.

(f) State and prove the Euler's theorem for a homogeneous function of three variables.

3. Answer any two questions of the following : $10 \times 2 = 20$

(a) (i) Prove that

$$\iiint \frac{dx dy dz}{x^2 + y^2 + (z-2)^2} = \pi \left(2 - \frac{3}{2} \log 3 \right),$$

extended over the sphere $x^2 + y^2 + z^2 \leq 1$. 6

(ii) Explain the term 'Differentiability' and 'Total Differential' for a function of two variables. 4

(b) (i) Find the constants a and b so that the surface

$ax^2 - byz = (a+2)x$ will be orthogonal to

the surface $4x^2y + z^3 = 4$ at the point

$(1, -1, 2)$. 4

(ii) Verify Stokes' theorem for $\vec{F} = (2y+z)\hat{i} +$

$(x-z)\hat{j} + (y-x)\hat{k}$, over the triangle ABC

cut from the plane $x + y + z = 1$ by the coordinate planes. 6

(c) (i) The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on

the ellipse that lie closest to and farthest from the origin. 6

(ii) Prove that $\iiint_V \vec{\nabla} \times \vec{B} dV = \iint_S \hat{n} \times \vec{B} dS$. 4

(d) (i) Using Lagrange multiplier method, prove the

inequality $\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$, $x \geq 0$, $y \geq 0$, $z \geq 0$. 6

(ii) Find the length of the arc of the parabola $(y-2)^2 = 16(x-1)$ measured from the vertex to an extremity of the latus rectum. 4