2023

5th Semester Examination MATHEMATICS (Honours)



Paper: C 11-T

[Partial Differential Equations and Applications]

[CBCS]

Full Marks: 60

Time: Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any ten questions:

 $2 \times 10 = 20$

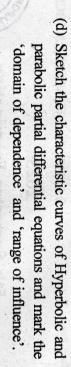
- (a) Find the partial differential equation by eliminating arbitrary constants a and b from $u = ae^{-b^2t} \sin bx$.
- (b) Verify that $u(x, t) = \frac{1}{\sqrt{4\pi kt}} e^{\frac{-x^2}{4kt}}$ is a solution of $u_t = ku_{xx}$.
- (c) Find the complete integral of the partial differential equation

$$\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) \left(z - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}\right) = 1.$$

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where
$$p = \frac{\partial z}{\partial x}$$
 and $q = \frac{\partial z}{\partial y}$ are compatible.

(f) Find the characteristic curves of the PDE

$$\frac{\partial^2 z}{\partial y^2} - y \frac{\partial^2 z}{\partial x^2} = 0.$$

(g) Solve:
$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$
.

(h) Find the particular solution of the PDE:

$$(D^2 - 2DD' + D'^2)z = xy$$
 where $D = \frac{\partial}{\partial x}$ and

$$D' \equiv \frac{\partial}{\partial v}.$$

- (i) Propose a problem of semi-infinite string with (a) free end and (b) fixed end.
- (j) A particle describes a curve $s = c \tan \psi$ with uniform speed ν . Find the acceleration indicating its direction.



(k) A particle describes a curve whose equation is $\frac{a}{r} = \theta^2 + b \text{ under a force to the pole. Find the law of force.}$

(l) A particle moves in an ellipse under the force μ/r^2 towards the focus. Find the velocity at any point of its path.

(m) Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \neq 0$. Then prove that the Cauchy problem $a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = 1$,

 $x, y \in \mathbb{R}$ with z(x, y) = x on ax + by = 1 has a unique solution.

(n) Show that all the surfaces of revolution, $z = f(x^2 + y^2)$ with the z-axis as the axis of symmetry, where f is an arbitrary function, satisfy the partial differential equation yp - xq = 0.

(o) Find the nature of the PDE: $(x^2 - y^2) \frac{\partial^2 z}{\partial x^2} +$

$$2(x^2 + y^2)\frac{\partial^2 z}{\partial x \partial y} + (x^2 - y^2)\frac{\partial^2 z}{\partial y^2} = 0 \quad \text{for } x > 0$$



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Group - B

2. Answer any *four* questions :

5×4=20

(a) Solve the equation $y^2 u_x^2 + x^2 u_y^2 = (xyu)^2$ by separation of variable method.

- (b) Solve $yz \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = xy$ and hence find the integral surface passing through $z^2 - y^2 = 1$, $x^2 - y^2 = 4$.
- (c) A particle moves describing an ellipse under a force respectively, prove that $v^2v_2^2 = v_1^2(2v_2^2 - v_1^2)$. ends of the latus rectum and major and minor axes to the centre, if ν , ν_1 , ν_2 are the velocities at the
- (d) Find the characteristics and reduce the equation to its canonical form

$$u_{xx} - 4u_{xy} + 4u_{yy} = e^{y}$$
.

- (e) If a planet were suddenly stopped in its orbit the planet's revolution. Sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of supposed circular, show that it would fall into the
- (f) A particle moves in a plane under a force towards a fixed centre, proportional to the distance. If the

(5)

path of the particle has two apsidal distance a, b (a > b), show that its equation can be written in

the form $u^2 = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}$

Group - C

 $10 \times 2 = 20$

3. Answer any two questions:

- (i) A curve is described by a particle having a a constant angle to the tangent. Show that the constant acceleration in a direction inclined at curve is an equiangular spiral.
- (ii) A machine gun of mass M_0 stands on a uniform rate with constant velocity u relative mass m, which is fixed horizontally at a to the gun. If the coefficient of sliding friction horizontal plane and contains a shot of total shot is all expended in time T, show that between the gun and the plane is µ, and the $u\log(1+m/M_0)-\mu gT$ the velocity of the gun is then
- 3 (i) Derive the Laplace equation for the distribution of gravitational potential.

$$x(y^2-z^2)p-y(z^2+x^2)q=z(x^2+y^2).$$
5+5

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(c) (i) Find the solution of the initial boundary value problem



$$u_t = ku_{xx}, \ 0 < x < l, \ t > 0,$$

 $u(0, t) = 0, \ t \ge 0$
 $u(l, t) = 0, \ t \ge 0,$

 $u(x, 0) = f(x), 0 \le x \le l$ using separation of variables.

- (ii) If a particle moves in a plane curve, prove that the sum of kinetic energy and the potential energy is constant when the force is conservative.
- (d) (i) If the orbit described by a particle under a central force to the origin be $r^n \cos n\theta = a^n$. Find the law of force.
 - (ii) Let z(x, y) be the solution of the first order partial differential equation $x \frac{\partial z}{\partial x} + (x^2 + y) \frac{\partial z}{\partial y}$ = z, $\forall x, y \in \mathbb{R}$ satisfying z(2, y) = y - q $y \in \mathbb{R}$. Then find the value of z(1, 2). 6+4