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PG CBCS
M.Sc. Semester-IV Examination, 2023
(Mathematics)
PAPER: MTM 405B
(OPERATIONAL RESEARCH MODELLING - II)

Full Marks: 25 **Time: 1 Hour**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **TWO** of the following questions: 2×2=4
   a) For an equipment, the reliability per 100 hours of operation has been estimated to be 0.999. what is the failure rate of the equipment? Calculate MTBF?
   b) $J = \int_1^3 y(3x - y)\,dx$ where $y(1) = 1, y(3) = \frac{9}{2}$. Find the curve along with the given curve is extremum. How many such curve exist.
   c) Define entropy function and explain its importance.
   d) State Pontryagin's maximum principle.

2. Answer any **TWO** of the following questions: 2×4=16
   a) Three generators one with a capacity of 100 kw and the other two with a capacity of 50 kw each are connected in parallel. Draw the reliability logic diagram if the required load is: (i) 100 kw (ii) 150 kw. Determine the reliability of both the arrangements if the reliability of each generator is 0.95
   b) Find that the entropy of the following probability distribution is

Event:	x_1	x_2	$\cdots$	x_i	$\cdots$	x_{n-1}	x_n
Probability:	$\frac{1}{2}$	$\frac{1}{2^2}$	$\cdots$	$\frac{1}{2^i}$	$\cdots$	$\frac{1}{2^{n-1}}$	$\frac{1}{2^{n-1}}$

   c) An electronic device has a failure rate of 500 failures per 10^6 hours. One identical stand-by unit is added to increase the reliability of the basic device. The operating time is 1000 hours. The failure rate of the sensing and switching element is 0.97. What will be the system reliability if the sensing and switching element is 100% reliable?
   d) Prove that the reliability function for random failure is an exponential distribution. How system reliability can be improved?

(P.T.O)

3. Answer any **ONE** of the following questions: 1×8=8

a) i) Prove that the entropy function $H(p_1, p_2, \dots, p_n)$ is continuous in p_k $\forall\, 0 \le p_k \le 1$.

ii) Find the least value of the integral $J = \int_0^1 \left[1 + \left(\frac{d^2x}{dt^2}\right)^2\right] dt$. Subject to the boundary conditions $x(0) = 0, x(1) = 1, \dot{x}(0) = 1, \dot{x}(1) = 1$.

2+6

b) i) Find the stationary path $x = x(t)$ for the functional $J = \int_0^1 \left[1 + \left(\frac{d^2x}{dt^2}\right)\right] dt$, where boundary conditions are $x(0) = 0, x(1) = \dot{x}(0) = \dot{x}(1) = 1$.

ii) What do you mean by "Mean time between failure" of an item. 6+2

[Internal Assessment – 05 Marks]

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