

PG CBCS
M.Sc. Semester-IV Examination, 2023
MATHEMATICS
PAPER: MTM 404B
(NON-LINEAR OPTIMIZATION)



Full Marks: 50

Time: 2 Hours

The figures in the right-hand margin indicate full marks.

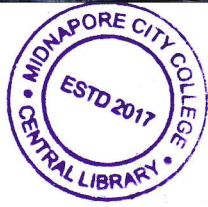
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **FOUR** of the following questions: 4×2=8
- Define Kuhn-Tucker saddle point problem.
 - What is Stochastic Programming Problem?
 - Define Nash equilibrium strategy and Nash equilibrium outcome in connection into Bi-matrix game.
 - What is degree of difficulty in connection with geometric programming?
 - Define Pareto optimal solution in a multi-objective non-linear programming problem.
 - State Karlin's constraint qualification.
2. Answer any **FOUR** of the following questions: 4×4=16
- State and prove the Fritz- John stationary point necessary optimality theorem.
 - State and prove weak duality theorem in connection with duality in non-linear programming.
 - Discuss the relationship between Minimization problem, Local Minimization problem, the Kuhn- Tucker stationary point problem, the Fritz-John Stationary point problem.
 - Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. Prove that θ is convex if and only if $\theta(x^2) - \theta(x^1) \geq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$
 - Define the following terms:
 - The (primal) quadratic minimization problem (QMP).
 - The quadratic dual (maximization) problem (QDP).
 - Prove that all strategically equivalent bi-matrix games have the same Nash equilibria.
3. Answer any **TWO** of the following questions: 2×8=16
- Solve by using Wolfe's method the following quadratic programming problem

$$\begin{aligned} \text{Max } z &= 2x_1 + 3x_2 - 2x_1^2 \\ \text{Sub. to } x_1 + 4x_2 &\leq 4 \\ x_1 + x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(P. T. O)

(1)



b) How do you solve the following geometric programming problem?

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

$$f(x) = \sum_{j=1}^n U_j(x) = \sum_{j=1}^N \left(c_j \prod_{i=1}^n x_i^{a_{ij}} \right)$$

$c_j > 0, x_i > 0, a_{ij}$ are real numbers, $\forall i, j$. Write the differences between polynomial & posynomial. 6+2

c) Using the chance-constrained programming technique to find an equivalent deterministic problem of the following stochastic programming problem of the following stochastic programming problem.

$$\begin{aligned} \text{minimize } F(x) &= \sum_{j=1}^n c_j x_j \\ \text{subject to } P[\sum_{j=1}^n a_{ij} x_j \leq b_i] &\geq p_i \\ x_i &\geq 0, i, j = 1, 2, \dots, n \end{aligned}$$

d) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem:

- (i) Complete optimal solution
- (ii) Pareto optimal solution
- (iii) Local Pareto optimal solution
- (iv) Weak Pareto optimal solution.

[Internal Assessment- 10 Marks]
