Total pages: 02

MCC/21/M.Sc./Sem-IV/MTM/1

PG CBCS
M.Sc. Semester-IV Examination, 2023
MATHEMATICS
PAPER: MTM 404B
(NON-LINEAR OPTIMIZATION)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **FOUR** of the following questions: 4×2=8
   a) Define Kuhn-Tucker saddle point problem.
   b) What is Stochastic Programming Problem?
   c) Define Nash equilibrium strategy and Nash equilibrium outcome in connection into Bi-matrix game.
   d) What is degree of difficulty in connection with geometric programming?
   e) Define Pareto optimal solution in a multi-objective non-linear programming problem.
   f) State Karlin's constraint qualification.

2. Answer any **FOUR** of the following questions: 4×4=16
   a) State and prove the Fritz- John stationary point necessary optimality theorem.
   b) State and prove weak duality theorem in connection with duality in non-linear programming.
   c) Discuss the relationship between Minimization problem, Local Minimization problem, the Kuhn- Tucker stationary point problem, the Fritz-John Stationary point problem.
   d) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. Prove that θ is convex if and only if $\theta(x^2) - \theta(x^1) \geqq \nabla\theta(x^1)(x^2 - x^1)$for each $x^1, x^2 \in \Gamma$
   e) Define the following terms:
      i) The (primal) quadratic minimization problem (QMP).
      ii) The quadratic dual (maximization) problem (QDP).
   f) Prove that all strategically equivalent bi-matrix games have the same Nash equilibria.

3. Answer any **TWO** of the following questions: 2×8=16
   a) Solve by using Wolfe's method the following quadratic programming problem

$$Max\ z = 2x_1 + 3x_2 - 2x_1^2$$
$$Sub.\ to\ x_1 + 4x_2 \leq 4$$
$$x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

(P. T. O)

b) How do you solve the following geometric programming problem?

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

$$f(x) = \sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N} \left(c_j \prod_{i=1}^{n} x_i^{a_{ij}} \right)$$

$c_j > 0, x_i > 0, a_{ij}$ are real numbers, $\forall i,\ j$. Write the differences between polynomial & posynomial. 6+2

c) Using the chance-constrained programming technique to find an equivalent deterministic problem of the following stochastic programming problem of the following stochastic programming problem.

$$minimize\ F(x) = \sum\nolimits_{j=1}^{n} c_j x_j$$

$$subject\ to\ P[\textstyle\sum_{j=1}^{n} a_{ij} x_j \le b_i] \ge p_i$$

$$x_i \ge 0, i, j = 1, 2, \dots, n$$

d) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem:
(i) Complete optimal solution (ii) Pareto optimal solution
(iii) Local Pareto optimal solution (iv) Weak Pareto optimal solution.

[Internal Assessment- 10 Marks]

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