MCC/22/M.SC./SEM.-III/MTM/1

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PG (CBCS)
M.SC. Semester- III Examination, 2023
MATHEMATICS
PAPER: MTM 302
(TRANFORMS AND INTEGRAL EQUATIONS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **FOUR** of the following questions: 4×2=8

a) Verify the final value theorem in connection with Laplace transform for the function t^3e^{-t}

b) Define wavelet function and analyze the parameters involved in it.

c) Define the inversion formula for Fourier transform of the function $f(x)$. What happens if $f(x)$ is continuous.

d) If $\bar{G}(x,y)$ be the two-dimensional Fourier transform of a function $G(x,y)$, then what is the Fourier inversion formula to get $G(x,y)$ from $\bar{G}(k,l)$.

e) If $F(p)$ denotes the Laplace transform of the function $f(t), t \geq 0$, state the conditions which $f(t)$ must satisfy so that $F(p)$ exists.

f) Define singular integral equation with an example.

GROUP-B

2. Answer any **FOUR** of the following questions: 4×4=16

a) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform.

b) Let $F(k)$ and $G(k)$ be the Fourier transforms of $f(x)$ and $g(x)$ respectively defined in $(-\infty,\infty)$. Show that the Fourier transform of $\int_{-\infty}^{\infty} f(u)g(x-u)du$, can be expressed in terms of the product $F(k)G(k)$. Hence prove that parseval's relation $\int_{-\infty}^{\infty} |f(k)|^2 dk = \int_{-\infty}^{\infty} |f(x)|^2 dx$.

c) If a and b are real constants, solve the following integral equation: $ax + bx^2 = \int_0^x \frac{y(t)}{(x-t)^{\frac{1}{2}}} dt$.

d) Find the Fourier transform of the function $f(x) = \begin{cases} 1, & |x| < 0 \\ 0, & |x| > 0 \end{cases}$. Hence evaluate $\int_0^{\infty} \frac{sinx}{x} dx$

e) Evaluate $L = \left\{\int_0^t \frac{\sin u}{u} du\right\}$ by the help of Initial Value theorem.

f) Solve $ty''(t) + 2y'(t) + ty(t) = 0$, subject to the condition $y(0+) = 1, y(\pi) = 0$.

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GROUP-C

3. Answer any **TWO** of the following questions: 2×8=16

a) (i) With help of the resolvent kernel, find the solution of the integral equation $y(x) = 1 + x^2 + \int_0^x \left(\frac{1+x^2}{1+t^2}\right) y(t)dt$.

(ii) Discuss the solution procedure of homogeneous procedure of homogeneous Fredholm integral equation of the second kind with degenerate kernel. 4+4

b) (i) Form an integral equation corresponding to the following differential equation $\frac{d^2y}{dx^2} + (1-x)\left(\frac{dy}{dx}\right) + e^{-x}y(x) = x^3 - 5x$, subject to conditions $y(0) = -3$ and $y'(0) = 4$.

(ii) If $L\{f(t)\} = F(p)$ which exists $Real(p) > \gamma$ and $H(t)$ is unit step function, then prove that for any $\alpha, L\{H(t-\alpha)f(t-\alpha)\} = e^{-p\alpha}F(p)$, which exists for $Real(p) > \gamma$. 5+3

c) i) Reduce the following BVP to an integral equation $y''(x) = -\lambda y(x)$ with boundary condition $y(0) = 0$, $y(1) = 0$.

ii)Define Degenerate kernel with example. 6+2

d) Solve the following boundary value problem in the half place $y > 0$, describe by

PDE: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, -\infty < x < \infty, y > 0$

BCS: $u(x,0) = f(x), -\infty < x < \infty$

u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$.

[Internal Assessment- 10 Marks]

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