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PG (CBCS)
M.SC. Semester- III Examination, 2023
MATHEMATICS
PAPER: MTM 301
(PARTIAL DIFFERENTIAL EQUATIONS AND GENERATED FUNCTIONS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **FOUR** of the following questions: 4×2=8

a) What are the main differences between an ODE and PDE?

b) Show by using weak maximum and weak minimum principle that the Dirichlet problem for the Poisson's equation has unique solution.

c) Define characteristic curve and base characteristics of a first order quasi linear PDE.

d) Classify the following equations

(i) $x^2U_{xx} - 2xyU_{xy} + y^2U_{yy} + xU_x + yU_y = 0$

(ii) $U_{xx} + xU_{yy} = 0, x > 0$.

e) Prove that for any continuous function $f(t)$, $\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$, where, $\delta(t)$ is dirac-delta function.

f) Define Robin boundary problem with an example.

GROUP-B

2. Answer any **FOUR** of the following questions: 4×4=16

a) Let $u(x,y)$ be an integral surface of the equation $a(x,y)u_x + b(x,y)u_y + u = 0$, where $a(x,y)$ and $b(x,y)$ are positive differentiable functions in the entire plane. Define $D = \{(x,y): |x| < 1, |y| < 1\}$.
(i) Show that if u be positive on the boundary of D, then it is positive at every point in D.
(ii) Suppose that u attains a local minimum (maximum) at a point $(x_0, y_0) \in D$. Find $u(x_0, y_0)$.

b) Solve the equation $\Delta u = 0$ in the disc $D = \{(x,y): x^2 + y^2 < a^2\}$ with the boundary condition $u = 1 + 3\sin\theta$ on the circle $r = a$.

c) Prove the followings for a continuous function f(t) and dirac-delta function δ(t):

(i) $\int_{-\infty}^{\infty} f(t-a)\delta(t)dt = f(a)$

(ii) $\delta(-t) = \delta(t)$.

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d) Prove that the solution we found by separation of variables for the vibration of a free string can be represented as a superposition of a forward and backward wave.

e) State and prove strong maximum principle.

f) Show that the Green function for the Laplace equation is symmetric.

GROUP-C

3. Answer any **TWO** of the following questions: 2×8=16

a) (i) Solve the following PDE:

$$(D^2 + DD' - 2D'^2)z = e^{x+y}.$$

(ii) Determine D'Alembert's formula for the Cauchy problem of the following homogeneous wave equation:

$$U_{tt} - c^2U_{xx} = 0$$
$$U(x,0) = f(x)$$
$$U_t(x,0) = g(x)$$

5+3

b) (i) Solve the following: $(D^2 + 5DD' + D'^2)z = 0$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

(ii) Check the validity of the maximum principle for the harmonic function $\frac{1-x^2-y^2}{1-2x+x^2+y^2}$ in the disc $\bar{D} = \{(x,y): x^2 + y^2 \leq 1\}$. 4+4

c) (i) Prove that the type of a linear second order partial differential equation in two variables is invariant under a change of co-ordinates.

(ii) Prove that Laplace operator is a self-adjoint operator. 5+3

d) Using energy method show that the solution is unique of the following problem:

$$U_{tt} - c^2U_{xx} = F(x,t), o < x < L, t > 0$$
$$U(0,t) = a(t), U(L,t) = b(t), t \geq 0$$
$$U(x,0) = f(x), 0 \leq x \leq L$$
$$U_t(x,0) = g(x), 0 \leq x \leq L$$

[Internal Assessment- 10 Marks]

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