

PG (CBCS)
M.SC. Semester- III Examination, 2023
MATHEMATICS
PAPER: MTM 301
(PARTIAL DIFFERENTIAL EQUATIONS AND GENERATED FUNCTIONS)
Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

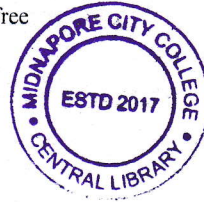
GROUP-A

1. Answer any **FOUR** of the following questions: 4×2=8
- What are the main differences between an ODE and PDE?
 - Show by using weak maximum and weak minimum principle that the Dirichlet problem for the Poisson's equation has unique solution.
 - Define characteristic curve and base characteristics of a first order quasi linear PDE.
 - Classify the following equations
 - $x^2U_{xx} - 2xyU_{xy} + y^2U_{yy} + xU_x + yU_y = 0$
 - $U_{xx} + xU_{yy} = 0, x > 0.$
 - Prove that for any continuous function $f(t)$, $\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$, where, $\delta(t)$ is dirac-delta function.
 - Define Robin boundary problem with an example.

GROUP-B

2. Answer any **FOUR** of the following questions: 4×4=16
- Let $u(x, y)$ be an integral surface of the equation $a(x, y)u_x + b(x, y)u_y + u = 0$, where $a(x, y)$ and $b(x, y)$ are positive differentiable functions in the entire plane. Define $D = \{(x, y): |x| < 1, |y| < 1\}$.
 - Show that if u be positive on the boundary of D , then it is positive at every point in D .
 - Suppose that u attains a local minimum (maximum) at a point $(x_0, y_0) \in D$. Find $u(x_0, y_0)$.
 - Solve the equation $\Delta u = 0$ in the disc $D = \{(x, y): x^2 + y^2 < a^2\}$ with the boundary condition $u = 1 + 3 \sin\theta$ on the circle $r = a$.
 - Prove the followings for a continuous function $f(t)$ and dirac-delta function $\delta(t)$:
 - $\int_{-\infty}^{\infty} f(t-a)\delta(t)dt = f(a)$
 - $\delta(-t) = \delta(t)$.

- d) Prove that the solution we found by separation of variables for the vibration of a free string can be represented as a superposition of a forward and backward wave.
 e) State and prove strong maximum principle.
 f) Show that the Green function for the Laplace equation is symmetric.

**GROUP-C**3. Answer any **TWO** of the following questions:

2×8=16

- a) (i) Solve the following PDE:

$$(D^2 + DD' - 2D'^2)z = e^{x+y}.$$

- (ii) Determine D'Alembert's formula for the Cauchy problem of the following homogeneous wave equation:

$$\begin{aligned} U_{tt} - c^2 U_{xx} &= 0 \\ U(x, 0) &= f(x) \\ U_t(x, 0) &= g(x) \end{aligned}$$

- b) (i) Solve the following:
- $(D^2 + 5DD' + D'^2)z = 0$
- where
- $D \equiv \frac{\partial}{\partial x}$
- and
- $D' \equiv \frac{\partial}{\partial y}$
- .
- 5+3

- (ii) Check the validity of the maximum principle for the harmonic function
- $\frac{1-x^2-y^2}{1-2x+x^2+y^2}$
- in the disc
- $\bar{D} = \{(x, y): x^2 + y^2 \leq 1\}$
- .
- 4+4

- c) (i) Prove that the type of a linear second order partial differential equation in two variables is invariant under a change of co-ordinates.

- (ii) Prove that Laplace operator is a self-adjoint operator.
- 5+3

- d) Using energy method show that the solution is unique of the following problem:

$$\begin{aligned} U_{tt} - c^2 U_{xx} &= F(x, t), 0 < x < L, t > 0 \\ U(0, t) &= a(t), U(L, t) = b(t), t \geq 0 \\ U(x, 0) &= f(x), 0 \leq x \leq L \\ U_t(x, 0) &= g(x), 0 \leq x \leq L \end{aligned}$$

[Internal Assessment- 10 Marks]
