MCC/22/M.SC./SEM.-II/MTM/1

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PG (CBCS)
M.SC. Semester-II Examination, 2023
MATHEMATICS
PAPER: MTM 206
(GENERAL TOPOLOGY)

Full Marks: 25 **Time: 1 Hour**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **TWO** questions from the following: $2\times2=04$

   a) Let X and X' denote a single set in the two topologies τ and τ' , respectively. Let $i: X' \to X$ be the identity function. Show that i is continuous if and only if τ' is finer than τ.

   b) Show that $\mathbb{R}^n$ and $\mathbb{R}$ are not homeomorphic if $n > 1$.

   c) Show that the order topology on $\mathbb{Z}_+$ is the discrete topology.

   d) What do you mean by topological imbedding? Illustrate with an example.

2. Answer any **TWO** questions from the following: $2\times4=08$

   a) Consider the set $Y = [-1,1]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y? Which are open in $\mathbb{R}$? 2+2

      i) $A = \left\{x: \frac{1}{2} \leq |x| < 1\right\}$,

      ii) $B = \left\{x: 0 < |x| < 1 \; and \frac{1}{x} \notin \mathbb{Z}_+\right\}$

   b) Let X and Y be topological spaces and $f: X \to Y$. Prove that f is continuous if and only if $f(\bar{A}) \subset \overline{f(A)}$, for every subset A of X.

   c) Let Y be an ordered set in the order topology. Let f,g:$X \to Y$ be continuous. Show that the set $\{x \in X | f(x) \leq g(x)\}$ is closed in X.

   d) Show that every compact Hausdorff space is normal.

3. Answer any **ONE** questions from the following: $1\times8=08$

   a) 3+5

      i) Show that the product of two Hausdorff spaces is Hausdorff.

      ii) Let $f: A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = \left(f_\alpha(a)\right)_{\alpha \in J}$, where $f_\alpha: A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Prove that f is continuous if and only if each function f_α is continuous.

   b) 3+5

      i) Show that a subspace of a regular space is regular.

      ii) Define Lindelof space. Give example to show that the product of two Lindelof space need not be Lindelof.

[Internal Assessment- 05 Marks]

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