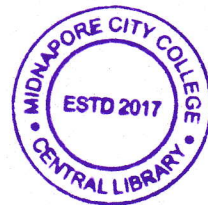


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PG (CBCS)
M.SC. Semester-II Examination, 2023
MATHEMATICS
PAPER: MTM 206
(GENERAL TOPOLOGY)



Full Marks: 25

Time: 1 Hour

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **TWO** questions from the following: 2×2=04
 - a) Let X and X' denote a single set in the two topologies τ and τ' , respectively. Let $i: X' \rightarrow X$ be the identity function. Show that i is continuous if and only if τ' is finer than τ .
 - b) Show that \mathbb{R}^n and \mathbb{R} are not homeomorphic if $n > 1$.
 - c) Show that the order topology on \mathbb{Z}_+ is the discrete topology.
 - d) What do you mean by topological imbedding? Illustrate with an example.

2. Answer any **TWO** questions from the following: 2×4=08
 - a) Consider the set $Y = [-1, 1]$ as a subspace of \mathbb{R} . Which of the following sets are open in Y ? Which are open in \mathbb{R} ? 2+2
 - i) $A = \left\{x: \frac{1}{2} \leq |x| < 1\right\}$,
 - ii) $B = \left\{x: 0 < |x| < 1 \text{ and } \frac{1}{x} \notin \mathbb{Z}_+\right\}$
 - b) Let X and Y be topological spaces and $f: X \rightarrow Y$. Prove that f is continuous if and only if $f(\overline{A}) \subset \overline{f(A)}$, for every subset A of X .
 - c) Let Y be an ordered set in the order topology. Let $f, g: X \rightarrow Y$ be continuous. Show that the set $\{x \in X \mid f(x) \leq g(x)\}$ is closed in X .
 - d) Show that every compact Hausdorff space is normal.

3. Answer any **ONE** questions from the following: 1×8=08
 - a) 3+5
 - i) Show that the product of two Hausdorff spaces is Hausdorff.
 - ii) Let $f: A \rightarrow \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha: A \rightarrow X_\alpha$ for each α . Let $\prod X_\alpha$ have the product topology. Prove that f is continuous if and only if each function f_α is continuous.
 - b) 3+5
 - i) Show that a subspace of a regular space is regular.
 - ii) Define Lindelof space. Give example to show that the product of two Lindelof space need not be Lindelof.

[Internal Assessment- 05 Marks]
