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PG CBCS

M.Sc. Semester-II Examination, 2023

MATHEMATICS

PAPER: MTM 205

(GENERAL THEORY OF CONTINNUM MECHANICS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as racticable.

1. Answer any **FOUR** of the following questions: 4×2=08

   a) Define perfect fluid.
   b) Write the differences between stream line and path line?
   c) Give examples of rotational and irrotational fluid flows.
   d) If the deformation of a body is defined by the displacement components $\boldsymbol{u_1} = \boldsymbol{k}(\boldsymbol{3X_1^2} + \boldsymbol{X_2}), \boldsymbol{u_2} = \boldsymbol{k}(\boldsymbol{X_2^2} + \boldsymbol{X_3})$ and $\boldsymbol{u_3} = \boldsymbol{k}(\boldsymbol{X_3} + \boldsymbol{X_1})$ where $\boldsymbol{k} > \boldsymbol{0}$. Compute the extension of a line element that passes through the point $(\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{1})$ in the direction $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.
   e) Find the complex potential due to a source.
   f) The velocity $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})$ of a fluid at a point $\boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ is given by $\boldsymbol{u} = -\frac{2xyz}{x^2+y^2}, \boldsymbol{v} = \frac{yz}{x^2+y^2}$ and $\boldsymbol{w} = \frac{z}{x^2+y^2}$. Find the rate at which density of the fluid at appoint P is decreasing in the flow field.

2. Answer any **FOUR** of the following questions: 4×4=16

   a) Define principal stress and principal direction of stress. Prove that all principal stresses are real.
   b) Show the equivalence between Eulerian and Lagrangian forms of equations of continuity.
   c) Find the image of a source with respect to a straight line.
   d) For the deformation defined by the equations $X_1 = \frac{1}{2}(x_1^2 + x_2^2), X_2 = \tan^{-1}\left(\frac{x_2}{x_1}\right), X_3 = x_3, x_1 \neq 0$, find the deformation gradient tensors in material forms. Hence show that the deformation is isochoric.
   e) The stress matrix at a point $P(x_i)$ in material is given by
   $$(T_{ij}) = \begin{pmatrix} x_1x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{pmatrix}$$
   Find the stress vector at the point $Q(1, 0, -1)$ on the surface $x_1 = x_2^2 + x_3^2$.
   f) Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principal directions of stress.

(P.T.O)

3. Answer any **TWO** of the following questions: 2×8=16

a) State and prove Kelvin's Minimum Energy theorem.

b) Derive the energy equation for Perfect fluid. 2+6

c) The stress tensor at appoint is given by

$$(T_{ij}) = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Determine the principal stresses and corresponding principal directions. Also check on the invariance of θ, θ_2 *and* θ_3. 2+4+2

d) Derive the basic elastic constants for isotropic elastic solid.

[Internal Assessment: 10 Marks]

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