

PG CBCS
M.Sc. Semester-II Examination, 2023
MATHEMATICS
PAPER: MTM 205

(GENERAL THEORY OF CONTINUUM MECHANICS)

Full Marks: 50

Time: 2 Hours

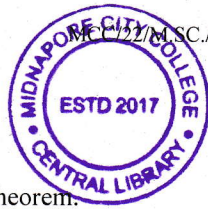
The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **FOUR** of the following questions: 4×2=08
- Define perfect fluid.
 - Write the differences between stream line and path line?
 - Give examples of rotational and irrotational fluid flows.
 - If the deformation of a body is defined by the displacement components $u_1 = k(3X_1^2 + X_2)$, $u_2 = k(X_2^2 + X_3)$ and $u_3 = k(X_3 + X_1)$ where $k > 0$. Compute the extension of a line element that passes through the point $(1, 1, 1)$ in the direction $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$.
 - Find the complex potential due to a source.
 - The velocity (u, v, w) of a fluid at a point $P(x, y, z)$ is given by $u = -\frac{2xyz}{x^2+y^2}$, $v = \frac{yz}{x^2+y^2}$ and $w = \frac{z}{x^2+y^2}$. Find the rate at which density of the fluid at appoint P is decreasing in the flow field.
2. Answer any **FOUR** of the following questions: 4×4=16
- Define principal stress and principal direction of stress. Prove that all principal stresses are real.
 - Show the equivalence between Eulerian and Lagrangian forms of equations of continuity.
 - Find the image of a source with respect to a straight line.
 - For the deformation defined by the equations $X_1 = \frac{1}{2}(x_1^2 + x_2^2)$, $X_2 = \tan^{-1}(\frac{x_2}{x_1})$, $X_3 = x_3$, $x_1 \neq 0$, find the deformation gradient tensors in material forms. Hence show that the deformation is isochoric.
 - The stress matrix at a point $P(x_i)$ in material is given by

$$(T_{ij}) = \begin{pmatrix} x_1 x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{pmatrix}$$
 Find the stress vector at the point $Q(1, 0, -1)$ on the surface $x_1 = x_2^2 + x_3^2$.
 - Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principal directions of stress.

(P.T.O)



(2)

3. Answer any **TWO** of the following questions:

- a) State and prove Kelvin's Minimum Energy theorem.
- b) Derive the energy equation for Perfect fluid.
- c) The stress tensor at a point is given by

2×8=16

2+6

$$(T_{ij}) = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Determine the principal stresses and corresponding principal directions. Also check on the invariance of θ , θ_2 and θ_3 .

2+4+2

- d) Derive the basic elastic constants for isotropic elastic solid.

[Internal Assessment: 10 Marks]
