Total pages: 02

PG (CBCS)
M.SC. Semester- I Examination, 2023
APPLIED MATHEMATICS
PAPER: MTM 105
(CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **FOUR** of the following questions: 2×4=8

a) Using Euler-Lagrange's equation, prove that the shortest distance between two points in a plane is a straight line.

b) Show that the following transformation is canonical $Q = \log\left(\frac{1}{q}\sin p\right), P = q\cot p$.

c) The Lagrangian for a coupled harmonic oscillator is given by

$$L = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2\right) - \frac{1}{2}\left(w_1^2 q_1^2 + w_2^2 q_2^2\right) + \alpha q_1 q_2$$

where α, w_1, w_2 are constants and q_1, q_2 are suitable coordinates. Find the Hamiltonian of the system.

d) What do you mean by non-inertial frame? Give an example of a non-inertial frame.

e) Show that for conservative holonomic dynamical system,

$$\frac{\partial L}{\partial \dot{q}_j} = \int \left(\frac{\partial L}{\partial q_j}\right) dt.$$

f) Write down the component and magnitude of the Coriolis force.

GROUP-B

2. Answer any **FOUR** of the following questions: 4×4=16

a) Obtain the curve for which the surface revolution is minimum.

b) The Hamiltonian of a dynamical system is given as $H = qp^2 - qp + bp$ where b is a constant. Solve the problem.

c) If a body in the northern hemisphere falls freely to the ground from a height h, show that it strikes the ground at $\frac{2}{3} wh \left(\frac{2h}{g_e}\right)^{\frac{1}{2}} \cos\lambda$ to the east, where w is the earth's angular velocity, g_e is the acceleration. Due to the combined effect of gravity and centrifugal force and λ is the latitude of the place.

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d) A uniform string of length l and negligible mass passes over a frictionless pulley. Two masses m_1 and m_2 are tied at two ends. Obtain the Lagrangian and write down the Lagrangian equation of motion.

e) Write down the Lagrange's equations when the Lagrangian has the following form $L = \dot{q}q - \sqrt{1-\dot{q}^2}$. Show that the following functional

$$J = \int_{x_0}^{x_1} \frac{(1+y^2)}{y'^2} dx$$

will be extremum if $y = sinh\,(c_1 x + c_2)$, where $c_1,\ c_2$ are arbitrary constant.

1+3

f) A body moves about a point Q under no forces. The principal moments of inertia at O being $3A, 5A$ and $6A$. Initially, the angular velocity has components $w_1 = n, w_2 = 0, w_3 = n$ about the corresponding principal axes. Show that at any time t,

$$w_2 = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right)$$

and that the body ultimately rotates about the mean axis.

GROUP-C

3. Answer any **TWO** of the following questions: 2×8=16

a) (i) Prove that $J = \int_{x_0}^{x_1} F(y, y', x)dx$ will be minimum only when $\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0$.

(ii) Find the extremum of the function $J = \int_0^{\log 2} (y + y'^2)dx$ subject to the boundary condition $y(0) = 0$ and $y(\log 2) = 1$. 5+3

b) Prove that Poisson bracket obeys the distributive law. If X, Y, Z are three dynamical variables, then prove the following:

$$(i)[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$$

$$(ii)[XY, Z] = Y[X,Z] + X[Y,Z].$$

2+4+2

c) Consider the following nonlinear dynamical system.

$$\frac{dx}{dt} = x^2 y - x^5, \frac{dy}{dt} = -y + x^2$$

Study the stability of the system at the origin.

d) What do you mean by inertial and non-inertial frame of references? Show that with respect to a uniformly rotating reference frame Newton's second law for a particle of mass acted upon by real force $\vec{F}$ can be expressed as

$\vec{F}_{eff} = \vec{F} - 2m\vec{w} \times \vec{V}_{rot} - m\vec{w} \times (\vec{w} \times \vec{r})$.

Assume that the origins of the inertial and non-inertial coordinates systems are coincident.

[Internal Assessment-10 Marks]

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