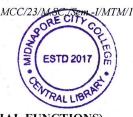
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# PG CBCS M.SC. Semester- I Examination, 2023 MATHEMATICS PAPER: MTM 103



# (ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS)

### Full Marks: 50

#### **Time: 2 Hours**

The figures in the right-hand margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

#### **GROUP-A**

#### 1. Answer any FOUR of the following questions:

2×4=8

- a) Prove that  $J_{-n}(z) = (-1)^n J_n(z)$  for integral n.
- b) Define Green's function of the differential operator L of the non-homogeneous differential equation: Lu(x) = f(x).
- c) Show that  $\int_{-1}^{1} P_n(z) dz = \begin{cases} 0, & n \neq 0 \\ 2, & n = 0 \end{cases}$  where the symbols have usual meaning.
- d) Find all the singularities of the following differential equation and then classify them:

(z - z<sup>2</sup>)w'' + (1 - 5z)w' - 4w = 0.

- e) Define orthogonal functions associated with Strum-Liouville problem.
- f) Prove that:  $F(-n; b, b; -z) = (1 + z)^n$  where F(a; b, c; z) denotes the hypergeometric function.

#### **GROUP-B**

#### 2. Answer any FOUR of the following questions:

4×4=16

- a) Show that  $J_0^2(z) + 2\sum_{n=1}^{\infty} J_n^2(z) = 1$  and prove that for real z,  $|J_0(z)| < 1$ , and  $|J_n(z)| < \frac{1}{\sqrt{2}}$ , for all  $n \ge 1$ .
- b) Show that  $1 + 3P_1(z) + 5P_2(z) + 7P_3(z) + \dots + (2n+1)P_n(z) = \frac{d}{dz}[P_{n+1}(z) + P_n(z)]$ , where  $P_n(z)$  denotes the Legendre's Polynomial of degree n.
- c) Consider the boundary value problem  $\frac{d^2y}{dx^2} + \lambda y = 0, 0 \le x \le \pi$  subject to  $y(0) = 0, y'(\pi) = 0$ . Find the eigen values and eigen functions of the problem.
- d) Using Green's function method, solve the following differential equation y'' y = x, subject to boundary conditions y(0) = y(1) = 0.

(P.T.O.)

- e) Prove that  $\int_{-1}^{1} P_m(z)P_n(z)dz = \frac{2}{2n+1}\delta_{mn}$ , where  $\delta_{mn}$  and  $P_n(z)$  are the Kroneker delta and Legendre's polynomial respectively.
- f) Show that  $\sqrt{\frac{\pi z}{2}} J_{\frac{3}{2}}(z) = \frac{1}{z} sinz cosz.$

# **GROUP-C**

## 3. Answer any <u>TWO</u> of the following questions:

#### 2×8=16

6 + 2

a) (i) Find the general solution of the homogeneous system

 $\begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$ (ii) Show that  $nP_n(z) = zP'_n(z) - P'_{n-1}(z)$ , where  $P_n(z)$  denotes the Legendre's Polynomial of degree n. 6+2

b) (i) Derive the Rodrigue's formula for Legendre's polynomial

(ii) Show that  $(n + 1)P_{n+1}(z) + nP_{n-1}(z) = (2n + 1)zP_n(Z)$ . 4+4

- c) (i) Prove that if f(z) is continuous and has continuous derivatives in [-1,1] then f(z) has unique Legendre's series expansion is given by  $f(z) = \sum_{n=0}^{\infty} C_n P_n(z)$  where  $P_n$ 's are Legendre Polynomials  $C_n = \frac{2n+1}{2} \int_{-1}^{1} f(z) P_n(z)$ , n = 1,2,3,...
  - (ii) Prove that  $\frac{d}{dz}[J_0(z)] = -J_1(z)$ .
- d) (i) Find the general solution of the ODE 2zw''(z) + (1+z)w'(z) kw = 0. (where k is a real constant) in series form for which values of k is there a polynomial solution?

(ii) Deduce the integral formula for hypergeometric function. 5+3

#### [Internal Assessment- 10 Marks]

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