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PG CBCS
M.SC. Semester- I Examination, 2023
APPLIEDMATHEMATICS
PAPER: MTM 102
(COMPLEX ANALYSIS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any FOUR of the following questions: **2×4=8**

a) State Morera's theorem.

b) Find residue of $f(z) = \frac{1}{2+z^2-2\cosh z}$ at $z = 0$.

c) Suppose z_1, z_2, z_3, z_4 are any four different points of a circle. Prove that the cross ratio is real.

d) Is it possible to evaluate the integral $\int_C f(z)dz$ where f(z)=(5z+2)/(z(z-2)) and C: |z|=1 using the single residue of $\frac{1}{z^2} f(\frac{1}{z})$ at z=0? Justify.

e) Find the Mobious transformation that maps 1,0, -1 to the respective points i, ∞,1

f) Find the order of the pole at $z = \frac{\pi}{4}$ of the function $f(z) = \frac{1}{cosz - sinz}$.

GROUP-B

2. Answer any FOUR of the following questions: **4×4=16**

a) Let $f(z) = z - 1$ and C is the arc from $z = 0$ to $z = 2\pi$ consisting the semicircle $z = 1 + e^{it}$. Find $\int_C f(z)dz$.

b) Find Laurent series expansion of $f(z) = \frac{1}{z-1} - \frac{1}{z-2}$ on $1 < |z| < 2$. Hence calculate the residue.

c) Use Rouche's Theorem, to determine the number of zeros of the polynomial $p(z) = z^{10} - 6z^9 - 3z + 1$.

d) Find the inverse of the bilinear transformation $f(z) = \frac{az+b}{cz+d}$ and show that this inverse function is also a bilinear. Also show that the determinants of both the transformations are the same.

(P.T.O.)

e) With the help of residue, find the inverse Laplace transformation f(t) of $F(s) = \frac{s}{(s^2+a^2)^2}$ $(a > 0)$.

f) Prove that the zeros of an analytic function are isolated.

GROUP-C

3. Answer any TWO of the following questions: **2×8=16**

a) (i) Prove that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$ corresponding to the transformation $u = u(x,y), v = v(x,y)$, where $\frac{\partial(u,v)}{\partial(x,y)}$ denotes the Jecobian of the transformation.

(ii) Prove that $\int_0^\infty \frac{x^4}{x^6-1} dx = \frac{\pi\sqrt{3}}{6}$. 3+5

b) Prove that the set of all bi-linear transformations forms a non-abelian group under the product transformation.

c) (i) Using the method of residues, evaluate $\int_0^\infty \frac{x^a}{(x^2+1)^2}$dx where -1<a<3 and $x^a = \exp(alnx)$.

(ii) Find the singular points of the function $\mathbf{z}|\mathbf{z}|$, if any. Justify your answer.

6+2

d) (i) Define the direct analytic continuation of an analytic function.

(ii) Use the Schwarz-Chritoffel transformation to arrive at the transformation $w = z^m$ $(0 < m < 1)$, which maps the half plane $y \geq 0$ onto the wedge $|w| \geq 0, 0 \leq \arg w \leq m\pi$ and transforms the point z=1 onto the point w=1. 2+6

[Internal Assessment- 10 Marks]

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