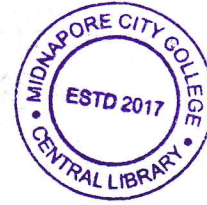


PG CBCS
M.SC. Semester- I Examination, 2023
APPLIED MATHEMATICS
PAPER: MTM 102
(COMPLEX ANALYSIS)



Full Marks: 50

Time: 2 Hours

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A1. Answer any **FOUR** of the following questions:

2×4=8

- a) State Morera's theorem.
- b) Find residue of $f(z) = \frac{1}{2+z^2-2 \cosh z}$ at $z = 0$.
- c) Suppose z_1, z_2, z_3, z_4 are any four different points of a circle. Prove that the cross ratio is real.
- d) Is it possible to evaluate the integral $\int_C f(z) dz$ where $f(z) = (5z+2)/(z(z-2))$ and $C: |z|=1$ using the single residue of $\frac{1}{z^2} f\left(\frac{1}{z}\right)$ at $z=0$? Justify.
- e) Find the Mobius transformation that maps $1, 0, -1$ to the respective points $i, \infty, 1$
- f) Find the order of the pole at $z = \frac{\pi}{4}$ of the function $f(z) = \frac{1}{\cos z - \sin z}$.

GROUP-B2. Answer any **FOUR** of the following questions:

4×4=16

- a) Let $f(z) = z - 1$ and C is the arc from $z = 0$ to $z = 2\pi$ consisting the semicircle $z = 1 + e^{it}$. Find $\int_C f(z) dz$.
- b) Find Laurent series expansion of $f(z) = \frac{1}{z-1} - \frac{1}{z-2}$ on $1 < |z| < 2$. Hence calculate the residue.
- c) Use Rouché's Theorem, to determine the number of zeros of the polynomial $p(z) = z^{10} - 6z^9 - 3z + 1$.
- d) Find the inverse of the bilinear transformation $f(z) = \frac{az+b}{cz+d}$ and show that this inverse function is also a bilinear. Also show that the determinants of both the transformations are the same.

(P.T.O.)



(2)

- e) With the help of residue, find the inverse Laplace transformation $f(t)$ of $F(s) = \frac{s}{(s^2+a^2)^2}$ ($a > 0$).
- f) Prove that the zeros of an analytic function are isolated.

GROUP-C**3. Answer any TWO of the following questions:****2×8=16**

- a) (i) Prove that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$ corresponding to the transformation $u = u(x,y), v = v(x,y)$, where $\frac{\partial(u,v)}{\partial(x,y)}$ denotes the Jacobian of the transformation.
- (ii) Prove that $\int_0^\infty \frac{x^4}{x^6-1} dx = \frac{\pi\sqrt{3}}{6}$. 3+5
- b) Prove that the set of all bi-linear transformations forms a non-abelian group under the product transformation.
- c) (i) Using the method of residues, evaluate $\int_0^\infty \frac{x^a}{(x^2+1)^2} dx$ where $-1 < a < 3$ and $x^a = \exp(alnx)$.
- (ii) Find the singular points of the function $z|z|$, if any. Justify your answer. 6+2
- d) (i) Define the direct analytic continuation of an analytic function.
- (ii) Use the Schwarz-Chritoffel transformation to arrive at the transformation $w = z^m$ ($0 < m < 1$), which maps the half plane $y \geq 0$ onto the wedge $|w| \geq 0, 0 \leq \arg w \leq m\pi$ and transforms the point $z=1$ onto the point $w=1$. 2+6

[Internal Assessment- 10 Marks]
