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PG CBCS
M.SC. Semester- I Examination, 2023
APPLIED MATHEMATICS
PAPER: MTM 101
(REAL ANALYSIS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any FOUR of the following questions: **2×4=8**

a) Let $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ -1, & x \in \mathbb{Q}^c \end{cases}$. Show that f is a measurable function on $\mathbb{R}$.

b) Given an example of a function f continuous on a closed interval $[a, b]$ but f is not a function of bounded variation on $[a, b]$.

c) Let f, g be two measurable functions. Show that the set $A = \{x: f(x) > g(x)\}$ is measurable.

d) Define Borel set. Also show that difference of two measurable sets is a measurable set.

e) Show that the set of all natural numbers is a null subset of $\mathbb{R}$.

f) Define Cantor set.

GROUP-B

2. Answer any FOUR of the following questions: **4×4=16**

a) Let f be a measurable function and let $f(x) = g(x)$ $a.e.$ Then $.g$ is measurable function.

b) Show that for any set A, $m^*(A) = m^*(A + x)$ where $A + x = \{y + x: y \in A\}$.

c) Let $f(x) = x^2 - 2x + 2$, $x \in [0,2]$. Express f as the difference of two monotone increasing functions on $[0,2]$. Hence show that f is a function of bounded variation on $[0,2]$.

d) Establish a necessary and sufficient condition for a function $f: [a, b] \to \mathbb{R}$ to be a function of bounded variation on $[a, b]$.

e) Show that every finite sum of real numbers can be expressed as the R-S integral over some interval.

f) If $f_n: X \to [0, \infty]$ is measurable for $n = 1,2,3, \ldots,$ and $f(x) = \sum_{n=1}^{\infty} f_n(x), x \in X$, then show that $\int f d\mu = \sum_{n=1}^{\infty} \int f_n d\mu$.

(P.T.O)

GROUP-C

3. Answer any TWO of the following questions: **2×8=16**

a) Show that if f is measurable, then the set $\{x : f(x) = \alpha\}$ is also measurable where $\alpha \in [-\infty, +\infty]$. Prove also that the constant functions are also measurable.

b) Show that the function $f(x) = x^{-\frac{1}{4}}$ on $S = [0, 1]$ is integrable and compute $\int_S f$.

c) (i) State and prove the Fatou's lemma. Give an example to show that strict inequality can occur in Fatou's lemma.

(ii) Let μ be a measure on a σ −algebra of subsets of X. Show that the outer measure μ^* induced by μ is countably subadditive. 4+4

d) (i) Construct a non-measurable subsets of $\mathbb{R}$.

(ii) Let $\{E_k\}$ be a sequence of measurable sets in X, such that $\sum_{k=1}^{\infty} \mu(E_k) < \infty$. Then prove that almost all $x \in X$ lie in at most finitely many of the sets E_k. 5+3

[Internal Assessment- 10 Marks]

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