

PG CBCS
M.Sc. Semester-I Examination, 2022
PHYSICS
PAPER: PHS 101
(METHODS OF MATHEMATICAL PHYSICS – I & CLASSICAL MECHANICS)
Full Marks: 40 **Time: 2 Hours**

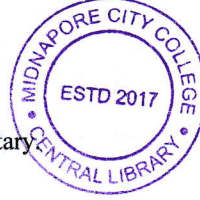
Write the answer for each unit in separate sheet

UNIT- PHS 101.1
METHODS OF MATHEMATICAL PHYSICS-I
GROUP-A

1. Answer any **TWO** from the following questions:

2×2=4

- a) Evaluate $\epsilon^{ijk}\epsilon_{ijk}$ in 3 dimensions.
- b) If $A = \begin{pmatrix} 0 & 1+2i \\ -1+2i & 0 \end{pmatrix}$ Show that $(I-A)(I+A)^{-1}$ is unitary.
- c) $\Psi_1 = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}; \Psi_2 = \begin{pmatrix} 3 \\ -5 \\ 2 \\ 2 \end{pmatrix}; \Psi_3 = \begin{pmatrix} 2 \\ -1 \\ 3 \\ 2 \end{pmatrix}$ Are these vectors linearly independent?
- d) If $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2 \theta (d\phi)^2$, Find $[22,1]$ in terms of r, θ and ϕ .



GROUP-B

2. Answer any **TWO** from the following questions:

2×4=8

- a) State and prove Schwarz inequality.
- b) If $A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$, Find A^{50} .
- c) Expand in Taylor's series $\frac{z-1}{z+1}$ about the point $z=1$.
- d) Using residue theorem, show that $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{1-2a \cos \theta + a^2} = \frac{2\pi a^2}{1-a^2}$ ($a^2 < 1$).

GROUP-C

3. Answer any **ONE** from the following questions:

1×8=8

- a) If $\omega_0 = 1, \omega_1 = x, \omega_2 = x^2$ use these functions to obtain three Hermite polynomials $H_0(x), H_1(x), H_2(x)$ by use of the Gram-Schmidt Orthogonalization process. (8)
- b) Evaluate $\int_0^{\infty} \frac{dx}{x^6+1}$ using residue theorem. (8)

P.T.O.

UNIT- PHS 101.2
CLASSICAL MECHANICS
GROUP-A

1. Answer any TWO of the following questions:

2×2=4

- a) If the Hamiltonian of a system does not involve time (t) explicitly, show that the generating functions $S(q, P, t)$ and $W(q, P)$ are then related to each other by the relation $S(q, P, t) = W(q, P) - \alpha_1 t$ where α_1 is a constant for separation of variable method.
- b) If $p(t+T) = p(t) + T[p, H] + \frac{T^2}{2!} [[p, H], H] + \frac{T^3}{3!} [[[p, H], H], H] + \dots$, then show that for a harmonic oscillator, $p(T) = -\omega X_0 \sin \omega T + \frac{P_0}{m} \cos \omega T$.
- c) Show that $\Delta \int_{t_1}^{t_2} \sum_k p_k q_k dt = \delta \int_{t_1}^{t_2} L dt + (L+H)[\Delta t]_{t_1}^{t_2}$.
- d) Obtain the Hamilton-Jacobi equation from time-dependent Schrodinger equation.

GROUP-B

2. Answer any TWO of the following questions:

2×4=8

- a) Show that, $\sum_{k=1}^n \left(T_{jk} \ddot{\xi}_k + V_{jk} \xi_k \right) = 0$ for Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium.
- b) Verify whether the following transformation is a contact transformation:
 $Q = \log \frac{\sin p}{q}$, $P = q \cot p$ Find the generating function F_1 .
- c) Define stable, unstable and natural equilibrium of a system.
- d) Show the gauge invariance of Lagrangian of a system.



GROUP-C

3. Answer any ONE of the following questions:

1×8=8

- (a) (i) Write down Jacobi's form of least action principle. Derive Hamilton's canonical equations using Hamilton's principle. (ii) $L = \frac{1}{2} q^2 - q\dot{q} + \dot{q}^2$, find p in terms of q, \dot{q} and calculate $\left[p, q^2 \right]$.
- (b) (i) If the Generating function be $S(q, P, t)$, then prove that Hamilton-Jacobi equation is to be $H + \frac{\partial S(q, P, t)}{\partial t} = 0$. (ii) A particle is thrown vertically upward with an initial velocity u against the attraction due to gravity. Write down the Hamilton-Jacobi for the motion and obtain general solution of the equation of motion.
