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PG CBCS
M.Sc. Semester-I Examination, 2022
PHYSICS
PAPER: PHS 101
(METHODS OF MATHEMATICAL PHYSICS – I & CLASSICAL MECHANICS)

Full Marks: 40 **Time: 2 Hours**

Write the answer for each unit in separate sheet

UNIT- PHS 101.1

METHODS OF MATHEMATICAL PHYSICS–I

GROUP-A

1. Answer any TWO from the following questions: **2×2=4**

a) Evaluate $\varepsilon^{ijk}\varepsilon_{ijk}$ in 3 dimensions.

b) If $A=\begin{pmatrix} 0 & 1+2i \\ -1+2i & 0 \end{pmatrix}$ Show that $(I-A)(I+A)^{-1}$ is unitary.

c) $\Psi_1=\begin{pmatrix}1\\3\\4\\2\end{pmatrix}; \Psi_2=\begin{pmatrix}3\\-5\\2\\2\end{pmatrix}; \Psi_3=\begin{pmatrix}2\\-1\\3\\2\end{pmatrix}$ Are these vectors linearly independent?

d) If $(ds)^2=(dr)^2+r^2(d\theta)^2+r^2\sin^2\theta(d\varphi)^2$, Find $[22,1]$ in terms of r, θ and φ.

GROUP-B

2. Answer any TWO from the following questions: **2×4=8**

a) State and prove Schwarz inequality.

b) If $A=\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, Find A^{50}.

c) Expand in Taylor's series $\frac{z-1}{z+1}$ about the point z=1.

d) Using residue theorem, show that $\int_0^{2\pi}\frac{\cos 2\theta d\theta}{1-2a\cos\theta+a^2}=\frac{2\pi a^2}{1-a^2}$ $(a^2<1)$.

GROUP-C

3. Answer any ONE from the following questions: **1×8=8**

a) If $\omega_0=1, \omega_1=x, \omega_2=x^2$ use these functions to obtain three Hermite polynomials $H_0(x), H_1(x), H_2(x)$ by use of the Gram-Schmidt Orthogonalization process.(8)

b) Evaluate $\int_0^{\infty}\frac{dx}{x^6+1}$ using residue theorem. (8)

P.T.O.

UNIT- PHS 101.2

CLASSICAL MECHANICS

GROUP-A

1. Answer any TWO of the following questions: **2×2=4**

a) If the Hamiltonian of a system does not involve time (t) explicitly, show that the generating functions $S(q,P,t)$ and $W(q,P)$ are then related to each other by the relation $S(q,P,t)=W(q,P)-\alpha_1 t$ where α_1 is a constant for separation of variable method.

b) If $p(t+T)=p(t)+T[p,H]+\frac{T^2}{2!}[[p,H],H]+\frac{T^3}{3!}[[[p,H],H],H]+......$, then show that for a harmonic oscillator, $p(T)=-\omega X_0 \sin\omega T+\frac{p_0}{m}\cos\omega T$.

c) Show that $\Delta\int_{t_1}^{t_2}\sum_k p_k \dot{q}_k dt=\delta\int_{t_1}^{t_2}\sum_k L dt+(L+H)[\Delta t]_{t_1}^{t_2}$.

d) Obtain the Hamilton-Jacobi equation from time-dependent Schrodinger equation.

GROUP-B

2. Answer any TWO of the following questions: **2×4=8**

a) Show that, $\sum_{k=1}^{n}\left(T_{jk}\ddot{\xi}_k+V_{jk}\xi_k\right)=0$ for Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium.

b) Verify whether the following transformation is a contact transformation: $Q=\log\frac{\sin p}{q}$, $P=q\cot p$ Find the generating function F_1.

c) Define stable, unstable and natural equilibrium of a system.

d) Show the gauge invariance of Lagrangian of a system.

GROUP-C

3. Answer any ONE of the following questions: **1×8=8**

(a) (i) Write down Jacobi's form of least action principle. Derive Hamilton's canonical equations using Hamilton's principle. (ii) $L=\frac{1}{2}\dot{q}^2-\dot{q}q+q^2$, find p in terms of q, $\dot{q}$ and calculate $\left[p,\dot{q}^2\right]$.

(b) (i) If the Generating function be $S(q,P,t)$, then prove that Hamilton-Jacobi equation is to be $H+\frac{\partial S(q,P,t)}{\partial t}=0$. (ii) A particle is thrown vertically upward with an initial velocity u against the attraction due to gravity. Write down the Hamilton-Jacobi for the motion and obtain general solution of the equation of motion.

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