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PG CBCS
M.SC. Semester-IV Examination, 2022
MATHEMATICS
PAPER: MTM 405B

(OPERATIONAL RESEARCH MODELLING-II)

Full Marks: 20 **Time: 1 Hour**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any two questions of the following: **2×2=4**

a) What do you mean by "Mean time between failure" of an item.

b) State pontryagin's maximum principle.

c) Define entropy function and explain its importance.

d) Find the curve X=X(t) which minimizes the functional $J=\int_0^1(\dot{X}^2+1)dt$ $X(0)=1$ and $X(1)=2$.

GROUP-B

2. Answer any two questions of the following: **4×2=8**

a) Write a brief note about control theory.

b) The two finite probability schemes are given by $(p_1, p_2, p_3, \ldots, p_n)$ and $(q_1, q_2, q_3, \ldots, q_n)$, with $\sum_{i=1}^n p_i=\sum_{i=1}^n q_i$, then show that $-\sum_{i=1}^n p_i log p_i \leq -\sum_{i=1}^n q_i \, log q_i$ with inequality holds if and only if $p_i=q_i$ for all i.

c) How many identical components each of which is 90% reliable over a period of 50 hours be used to obtain a 99.99% parallel redundancy system over 50 hours. If we want to obtain the system reliability over a period of 100 hours, how many components should be added?

d) Define the joint conditional entropies. Prove that H(X, Y)≤ H(X) + H(Y) with equality iff X and Y are independent.

[P. T. O]

GROUP-C

3. Answer any <u>one</u> questions of the following: **8×1=8**

a) i) Prove that the reliability function for random failure is an exponential distribution.

ii) In a system, there are n number of components connected in parallel with reliability $R_i(t)$; i=1,2…,n. Find the reliability of the system. If $R_1(t) = R_2(t) = \cdots = R_n(t) = e^{(-\lambda t)}$, λ is the failure rate then find the expression for system reliability.

4+4

b) A transmitter has a character consisting of five letters $\{x_1, x_2, \ldots\ldots, x_5\}$ and the receiver has a character consisting of four letters $\{y_1, y_2, y_3, y_4\}$. The joint probability for the communication is given below.Determine the entropies H(X), H(Y) and H(X, Y).

$p(x_i, y_j)$	y_1	y_2	y_3	y_4
x_1	0.25	0	0	0
x_2	0.10	0.30	0	0
x_3	0	0.05	0.10	0
x_4	0	0	0.05	0.10
x_5	0	0	0.05	0

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