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PG CBCS
M.SC. Semester-IV Examination, 2022
MATHEMATICS
PAPER: MTM 404B
(NONLINEAR OPTIMIZATION)

Full Marks: 40 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1.Answer any four questions of the following: **4×2=8**

a) Define posynomial and polynomial in connection with geometric programming problem with example.

b) Define bi-matrix game with example.

c) Write the advantages of geometric programming problem.

d) Define Pareto optimal solution in a multi-objective non-linear programming problem.

e) State Kuhn-Tucker stationary point necessary optimality theorem.

f) What are the basic differences between the necessary criteria and sufficient criteria of Fritz-John saddle point problem.

GROUP-B

2. Answer any four questions of the following: **4×4=16**

a) Prove that all strategically equivalent bi-matrix game have the same Nash equilibrium.

b) Derive the Kuhn-tucker condition for quadratic programming problem.

c) Find the minimum value of $f(X)$ by geometric programming $Min\, f(X) = 7x_1x_2^{-1} + 3x_2x_3^{-2} + 5x_1^{-3}x_2x_3 + x_1x_2x_3$.

d) State and Prove weak duality theorem in connection with duality in non-linear programming.

e) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset \mathbb{R}^n$. θ is convex if and only if $\theta(x^2) - \theta(x^1) \geq \nabla\theta(x^1)(x^2 - x^1) for\ each\ x^1, x^2 \in \Gamma$. Prove it.

[P. T. O]

f) Write the relationship among the solutions of minimization Problem (MP), local minimization problem (LMP), the Fritz-John stationary point problem (FJP), the Fritz-John saddle point problem (FJSP), the Kuhn-Tucker saddle point problem.

GROUP-C

3. Answer any <u>two</u> questions of the following: **8×2=16**

a) How do you solve the following geometric programming problem?

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

$f(x) = \sum_{j=1}^{N} U_j(x) = \sum_{j=1}^{N}(c_j \prod_{i=1}^{n} x_i^{a_{ij}})$

$c_j > 0, x_i > 0, a_{ij}$ are real numbers, $\forall\, i, j$.

b) Using the chance-constrained programming technique to find an equivalent deterministic LPP of the following stochastic programming problem.

$$Minimize\ F(x) = \sum_{j=1}^{n} c_j x_j$$

$$subject\ to\ P[\sum_{j=1}^{n} a_{ij}x_j \leq b_i] \geq p_i$$

$$x_j \geq 0, i, j = 1,2, \dots, n$$

When b_i is a random variable and p_i is a specified probability.

c) Solve the quadratic programming problem using Wolfe's method

$$Maximize\ z = 2x_1 + 3x_2 - 2x_1^2$$

$$subject\ to\ x_1 + 4x_2 \leq 4$$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

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