Total page: 03

PG CBCS

M.Sc. Semester-IV Examination, 2022

MATHEMATICS

PAPER: MTM 403

(MAGNETO HYDRO-DYNAMICS & STOCHASTIC PROCESS & REGRESSION)

Full Marks: 40 **Time: 2 Hours**

Write the answer for each unit in separate sheet

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

MTM 403.1: MAGNETO HYDRO-DYNAMICS

GROUP-A

1. Answer any two question: **2×2=4**

a) Define the term magnetic diffusivity.

b) State Ferraro's law of isorotation.

c) Write down the statement of Alfven's theorem

d) Define magnetic pressure and write its significance for the motion of conducting fluid.

GROUP-B

2. Answer any two questions: **2×4=8**

a) Write down the basic equations of mageto-hydrodynamics and hence deduce the magnetic induction equation in MHD flows.

b) Prove that in a steady non-uniformly rotating star, the angular velocity must be constant over the surface traced out by the rotation of the magnetic lines of force about the magnetic field axis.

c) Define magnetic energy and further, find the rate of change of magnetic energy in magneto-hydrodynamic.

d) Deduce magnetic induction equation.

GROUP-C

3. Answer any one questions: **8×1=8**

a) Define Couette flow. Give the mathematical formulation of mageto-hydrodynamic Couette flow and derive its velocity and magnetic field expression.

[P. T. O]

b) A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate z = - L (lower) and a horizontal infinitely long non-conducting plate z = L (upper). Assume that a uniform magnetic field H0 acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field.

MTM 403.2: STOCHASTIC PROCESS & REGRESSION

GROUP-A

1. Answer any two question: **2x2=4**

a) Define doubly stochastic matrix.

b) What is Ergodic process?

c) Define multiple correlation and partial correlation.

d) Write the transition matrix for the problem of random walk between reflecting barriers.

GROUP-B

2. Answer any two questions: **2×4=8**

a) Prove that for a Markov chain, state j is persistent iff $\sum_{n=0}^{\infty} p_{jj}{}^{(n)} = \infty$.

b) State and prove Chapman-Kologorov equation.

c) Derive the equation of the plane of regression containing three variables.

d) Let $\{{}^1/_n : n \geq 0\}$ be a Markov chain with three states 0,1,2 and with transition matrix $\begin{pmatrix} {}^3/_4 & {}^1/_4 & 0 \\ {}^1/_4 & {}^1/_2 & {}^1/_4 \\ 0 & {}^3/_4 & {}^1/_4 \end{pmatrix}$ and the initial distribution $P(X_0 = i) = {}^1/_3, i = 0, 1, 2$. Find

(i) $P(X_2 = 2, X_1 = 1 | X_0 = 2)$

(ii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$

GROUP-C

3. Answer any one questions: **1×8=8**

a) State birth and death process. Find the differential-difference equation for birth and death process. 2+6

b) Establish First Entrance Theorem. Consider the Markov chain on the state space $\{1, 2, 3, 4\}$ with transition probability matrix $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Draw the graph corresponding to this Markov chain. Identify the states as transient, persistent, ergodic. 3+1+4

*****