## PG CBCS

M.SC. Semester-III Examination, 2022

MATHEMATICS
PAPER: C-MTM 304
(DISCRETE MATHEMATICS)
Full Marks: 40
Time: 2 Hours

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions from the following:
$4 \times 2=08$
a) State Hand Shaking Lemma.
b) Define bipartite graph and give an example.
c) Define Poset.
d) Define chain and anti-chain with an example.
e) What do you know by eccentricity of a graph?
f) Find the language for the regular expression given below:

$$
(a+b) *(a+b b)
$$

2. Answer any FOUR questions from the following:
a) Show that the set $\mathbb{N}$ of all natural numbers under divisibility relation forms a poset.
b) In the Boolean algebra $(B,+, ., ')$, express the Boolean function $f(x, y, z)=$ $(x+y)(x+z)+y+z^{\prime}$ in its disjunctive normal form.
c) Prove that a connected graph with n vertices and $(\mathrm{n}-1)$ edges is a tree.
d) Define Planar graph and prove that the graph $K_{3,3}$ (Kuratowski's second graph) is non-planar.
e) Write down Huntington Postulates.
f) Define finite-state machine (FSM). Let M be the FSM with state table appearing as

|  | f |  | g |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{S} \sum$ | a | b | a | b |
| $s_{0}$ | $s_{1}$ | $s_{0}$ | 1 | 0 |
| $s_{1}$ | $s_{3}$ | $s_{0}$ | 1 | 1 |
| $s_{2}$ | $s_{1}$ | $s_{2}$ | 0 | 1 |
| $s_{3}$ | $s_{2}$ | $s_{1}$ | 0 | 0 |

(i) Find the input set $\sum$, the set set $S$, the output set $O$, and initial state of $M$.
(ii) Draw the state diagram of M .

## 3. Answer any TWO questions from the following:

$2 \times 8=16$
a) State the principle of inclusion-exclusion. Use the principle of inclusionexclusion, find the total number of integers between 1 and 1000 which are neither perfect squares nor perfect cubes.
$2+6$
b) Prove by mathematical induction $3+33+333+\ldots .+33 \ldots \ldots .3=\left(10^{n+1}-\right.$ $9 n-10) / 27$. Draw a full adder using half adder. $6+2$
c) Define phrase-structure grammar. Describe the classification scheme of phrase-structure grammar introduced by Noam Chomsky.
$2+6$
d) i) Determine the generating function of the following sequences:

$$
f_{r}=\frac{r(r+1)}{2},(r>0)
$$

ii) Let $G$ is a r- regular graph where $r$ is odd. Show that $G$ has even number of vertices. Again show that the number of edges of $G$ is multiple of $r$.

