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PG CBCS

M.Sc. Semester-III Examination, 2022

MATHEMATICS

PAPER: MTM 302

(INTEGRAL TRANSFORMS AND INTEGRAL EQUATIONS)

Full Marks: 40 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions: **4×2=8**

a) State Bromwich's integral formula concerning on inverse Laplace transform.

b) Define Fourier transform and state the conditions of existence of the transform.

c) Define eigen value and eigen vector in terms of an integral equation.

d) Define the inversion formula for Fourier cosine transform of the function $f(x)$. What happens if $f(x)$ is continuous.

e) What do you mean by Fredholm alternative in integral equation?

f) Verify the final value theorem in connection with Laplace transform for the function t^3e^{-t}.

2. Answer any FOUR questions: **4×4=16**

a) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform. 1+1+2

b) Prove that the Fourier transform of $\frac{1}{x}$ is $i\sqrt{\frac{\pi}{2}}\text{sgn}\,(\alpha)$, where sgn is signum function.

c) Reduce the boundary value problem $\frac{d^2y}{dx^2} + \lambda xy = 1, 0 \leq x \leq l$ with boundary conditions $y(0) = 0, y(e) = 1$ to an integral equation and find its Kernel.

d) Evaluate $L\{J_0(t)\}$ by the help of initial value theorem, $J_0(t)$ is the zeroth order of Bessel's function.

e) If the Fourier transform of $f(x)$ is $\frac{\alpha}{1+\alpha}$, α being the transform parameter, then find $f(x)$.

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f) Find the characteristic numbers and eigen functions of the homogeneous integral equation $y(x) = \lambda \int_0^{\pi} (\cos^2 x \cos 2t + \cos 3x \cos^3 t) y(t)\ dt$.

3. Answer any <u>TWO</u> questions: **2×8=16**

a) i) Find the resolvent kernel of the following integral equation and then solve it:

$u(x) = f(x) + \lambda \int_0^x e^{x-t} u(t) dt$

ii) Evaluate $\left\{\int_0^t J_0(s) J_1(t-s) ds\right\}$ 5+3

b) i) State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{ab(a+b)},\ a, b > 0.$

ii) Define the term convolution on Fourier transform. (4+2)+2

c) i) Solve $ty''(t) + y'(t) + 4ty(t) = 0$, subject to the conditions $y(0) = 3$, $y'(0) = 0$.

ii) Under certain conditions (to be specified by you), convert the following integral equation $f(x) = \int_0^x k(x,t) y(t) dt$ into the Volterra integral equation of 2nd kind. 5+3

d) Solve the following boundary value problem in the half plan $y > 0$, described by PDE: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $-\infty < x < \infty, y > 0$ with boundary conditions $u(x, 0) = f(x), -\infty < x < \infty$. u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$.

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