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PG (CBCS)

M.SC. Semester-III Examination, 2022

MATHEMATICS

PAPER: MTM 301

(PARTIAL DIFFERENTIAL EQUATIONS AND GENERALIZED FUNCTIONS)

Full Marks: 40 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions from the following: **4×2=8**

a) Define Cauchy problem for a first order Quasi-linear partial differential equation.

b) Show that $\delta(-t) = \delta(t)$ where δ is the Dirac delta function.

c) Show by using weak maximum and weak minimum principle that the Dirichlet problem for the Poisson's equation has unique solution.

d) Give an example of a harmonic function in a domain D which neither a maximum value nor a minimum value in D.

e) Define domain of dependence of the Cauchy problem for the wave equation.

f) Define adjoint operator of partial differential operator.

2. Answer any FOUR questions from the following: **4×4=16**

a) Find the canonical form of the equation: $U_{xx} + 4U_{xy} + U_x = 0$.

b) Solve the equation $\Delta u = 0$ in the disc $D = \{(x,y): x^2 + y^2 < a^2\}$ with the boundary condition $u = 1 + 3sin\theta$ on the circle r=a.

c) Prove that a Laplace operator is a self-adjoint operator.

d) Find the general solution of $(D^2 + D + D')z = x^2y$.

e) Let $u \in C^2(D)$ be a function satisfying the mean value property in D. Show that u is harmonic in D.

f) Define D'Alembert formula for the following Cauchy problem:

$$u_{tt} - u_{xx} = xt, -\infty < x < \infty, t > 0$$
$$u(x,0) = 0, -\infty < x < \infty$$
$$u_t(x,0) = e^x, -\infty < x < \infty.$$

(P.T.O.)

3. Answer any TWO questions from the following: **2×8=16**

a) (i) Solve the following Cauchy problem:

$$U_{tt} - 9U_{xx} = e^{x} - e^{-x}, -\infty < x < \infty, t > 0$$
$$U(x,0) = x, -\infty < x < \infty$$
$$U_t(x,0) = \sin x - \infty < x < \infty.$$

(ii) Consider the Cauchy problem

$$U_{tt} - U_{xx} = 0, -\infty < x < \infty, t > 0.$$

$$U(x,0) = f(x) = \begin{cases} 0, & -\infty < x < -1 \\ x+1, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0, & 1 < x < \infty \end{cases}$$

$$U\_t\ (x,0) = g(x) = \begin{cases} 0 & -\infty < x < -1 \\ 1 & -1 \le x \le 1 \\ 0 & 1 < x < \infty \end{cases}$$

Evaluate U at the point $\left(1, \frac{1}{2}\right)$. 5+3

b) (i) Find the derivative of the Heaviside unit step function.

(ii) Establish the d'Alembert's formula of the Cauchy problem for the non-homogeneous wave equation. 2+6

c) (i) State and prove the strong maximum principle.

(ii) Establish the Laplace equation in polar coordinates. 5+3

d) Consider the equation $u_{xx} - 2u_{xy} + 4e^{y} = 0, y > 0$

(i) Find the canonical form of the above equation.

(ii) Find the general solution u(x, y) of the equation.

(iii) Find the solution u (x, y) which satisfies $u(0,y) = f(y) and\ u_x(0,y) = g(y)$. 4+2+2

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