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PG CBCS

M.Sc. Semester-II Examination, 2022

(Mathematics)

PAPER: MTM 206

(GENERAL TOPOLOGY)

Full Marks: 20 **Time: 1 Hour**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any <u>two</u> questions: **2×2=4**

a) If Y is a subspace of X and Z is a subspace of Y, then show that Z is a subspace of X.

b) Give an example to show that a subspace of a normal space need not be normal.

c) Let X and X' denote a single set in the two topologies τ and τ', respectively. Let $i: X' \to X$ be the identity function. Show that i is continuous if and only if τ' is finer than τ.

d) Show that $\mathbb{R}^n$ and $\mathbb{R}$ are not homeomorphic if $n > 1$.

2. Answer any <u>two</u> questions: **2×4=8**

a) Show that the topologies $(\mathbb{R}, \tau_l)$ and $(\mathbb{R}, \tau_k)$ are not comparable.

b) Consider the set $Y = [-1,1]$ as a subspace of $\mathbb{R}$. Which of the following sets are open in Y? Which are open in $\mathbb{R}$?

(i) $A = \left\{x: \frac{1}{2} \leq |x| < 1\right\}$,

(ii) $B = \left\{x: 0 < |x| < 1 \; and \frac{1}{x} \notin \mathbb{Z}_+\right\}$.

c) If L is a straight-line in the plane, describe the topology L inherits as a subspace of $\mathbb{R}_l \times \mathbb{R}$ and as a subspace of $\mathbb{R}_l \times \mathbb{R}_l$. In each case it is a familiar topology.

d) In the finite complement topology on $\mathbb{R}$, to what point or points does the sequence $x_n = \frac{1}{n^2}$ converge?

[P.T.O]

3. Answer any one questions: **1×8=8**

a) (i) Show that the product of two Hausdorff spaces is Hausdorff.

(ii) Let $f: A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = \left(f_\alpha(a)\right)_{\alpha \in J}$, where $f_\alpha: A \to X_\alpha$ for each α. Let $\prod X_\alpha$ have the product topology. Prove that f is continuous if and only if each function f_α is continuous.

[3+5]

b) (i) Let $p: X \to Y$ be a closed continuous surjective map such that $p^{-1}(\{y\})$ is compact for each $y \in Y$. Show that if X is regular, then so is Y.

(ii) Show that if X has the discrete topology, then X is totally disconnected. Does the converse hold? **[3+5]**

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