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PG CBCS

M.Sc. Semester-II Examination, 2022

(Mathematics)

PAPER: MTM 202

(NUMERICAL ANALYSIS)

Full Marks: 40 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any four questions: **4×2=8**

a) Express the polynomial $x^4 - 5x^3 + 7x$ in terms of Chebyshev polynomials.

b) Find whether the following function is spline or not?

$$f(x) = \begin{cases} -x^2 - 2x^3, & x \in [-1,0] \\ -x^2 + 2x^3, & x \in [0,1] \end{cases}$$

c) State the minimax principle of polynomial interpolation.

d) Compare Gaussian quadrature and Monte-Carlo method to find integration.

e) Compare direct and iteration method to solve a system of linear equations.

f) What do you mean by single step and multi-step methods to solve an ODE ?

2. Answer any four questions: **4×4=16**

a) Analyze the stability of fourth order Runge-Kutta method for initial value ODE.

b) Use least squares method to fit the line $y = a + bx$ based on the sample $(2,1), \left(\frac{1}{6}, -\frac{5}{6}\right), \left(-\frac{3}{2}, -2\right)$ and $\left(-\frac{1}{3}, -\frac{2}{3}\right)$. Estimate the total error.

c) Find the value of $\int_0^1 \frac{1}{1+x^2} dx$ by Gauss-Legendre four points quadrature formula. Also, calculate the absolute errors.

d) Using Milne's predictor-corrector formula find $y(0.4)$ for the following IVP $\frac{dy}{dx} = x^2 - y;\ y(0) = 1$ with step length h=0.1.

e) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(0,t) = 0, u(1,t) = 2t$ and initial condition $u(x,0) = \frac{1}{2}x$.

[P.T.O]

f) Deduce 4-point Gauss-Legendre quadrature formula.

3. **Answer any <u>two</u> questions:** **2×8=16**

a) Describe power method for least eigenvalue. Find the largest eigenvalue in magnitude and corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 1 & 4 & 2 \\ -2 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}.$$ **[3+5]**

b) Describe the Crank-Necolson implicit method to solve the equation: $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(0,t) = f_1(t), u(1,t) = f_2(t)$ and the initial conditions $u(x,0) = g(x)$. **[5+3]**

c) (i) Discuss Milne's predictor-corrector formula to find the solution of $y' = f(x\, y), y(x_0) = y_0$.

(ii) Find the value of $y(0.20)$ for the initial value problem $\frac{dy}{dx} = y^2 sinx$, $y(0) = 1$ with step length $h = 0.25$ using Milne's method. **[3+5]**

d) Describe Lagrange's bivariate interpolation method. Hence find the value of $f(0.25, 0.75)$ from the data $f(0,0) = 1, f(1,0) = 1.732051, f(0,1) = 1.414214, f(1,1) = 2$. **[4+4]**

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