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M.SC. Semester-I Examination, 2022

DEPARTMENT OF MATHEMATICS

PAPER: MTM 105

(CLASSICAL AND NON-LINEAR DYNAMICS)

Full Marks: 40 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions from the following: **4×2=08**

a) Using Euler-Lagrange's equation, prove that the shortest distance between two points in a plane is a straight line.

b) A particle is constrained to move on the plane curve $xy = c$ under the incluence of gravity only. Obtain its Lagrange's equation of motion.

c) State the basic postulate of special theory of relativity.

d) Define Virtual work and generalized coordinates of a dynamical system.

e) Show that the following transformation is canonical $P = \frac{1}{2}(p^2 + q^2)$, $Q = tan^{-1}\left(\frac{q}{p}\right)$.

f) Prove that the gain of Kinetic energy of a moving particle is equal to the work done by the particle.

2. Answer any FOUR questions from the following: **4×4=16**

a) Find the Lagrange's equation of motion for a pendulum in spherical polar coordinates, of length l.

b) Show that the transformation

$$Q = \log(1 + \sqrt{q}\cos p), P = 2\sqrt{q}(1 + \sqrt{q}\cos p)\sin p$$

is canonical. Find the generating function $G(q, Q)$.

Then show that

(i) the transformation is canonical, and

(ii) the generating function of this transformation can be put in the form $F = -(e^Q - 1)^2 \tan p$. 2+2

c) Define the degrees of freedom of a dynamical system. Suppose a particle is constraint to move on a circular plane $x^2 + y^2 = a^2$. Then what would be the degree of freedom of the particle?

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d) Given a mass-spring system consisting of a mass m and a linear spring of stiffness k hanging from a fixed point. Find the Lagrangian equation of motion for the system.

e) Write down the Lagrange's equations when the Lagrangian has the following form $L = \dot{q}q - \sqrt{1-\dot{q}^2}$. Show that the following functional $J = \int_{x_0}^{x_1} \frac{(1+y^2)}{y'^2} dx$ will be extremum if $y = \sinh(c_1 x + c_2)$, where c_1, c_2 are arbitrary constant. 1+3

f) What is the effect of the Coriolis force on a particle falling freely under the action of gravity.

3. Answer any <u>TWO</u> questions from the following: **2×8=16**

a) State Hamilton's principle. Use it to derive the Lagrange's equation of motion for conservative system. 2+6

b) Prove that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation. Prove that $E = mc^2$, in relativistic mechanics.

c) Define cyclic coordinates for a dynamical system. For a dynamical system

$$T = \frac{1}{2}(1+2k)\dot{\theta}^2 + 2\dot{\theta}\dot{\varphi} + \dot{\varphi}^2;\ V = \frac{1}{2}n^2\{(1+k)\theta^2 + \varphi^2\}$$

θ, φ are constants, n, k are positive constants. Write down the Lagrange's equation of motion and deduce that

$$(\ddot{\theta} - \ddot{\varphi}) + n^2\left(\frac{1+k}{k}\right)(\theta - \varphi) = 0$$

and if $\theta = \varphi, \dot{\theta} = \dot{\varphi}$ at $t = 0$ then $\theta = \varphi$ for all t. 1+7

d) Prove that Poisson bracket obeys the distributive law. If X, Y, Z are three dynamical variables, then prove the following:

(i) $[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$

(ii) $[X,Y,Z] = Y[X,Z] + X[Y,Z]$. 2+4+2

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