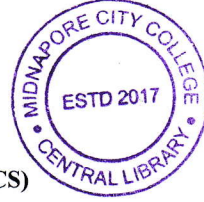


PG CBCS
M.SC. Semester-I Examination, 2022
DEPARTMENT OF MATHEMATICS
 PAPER: MTM 105
(CLASSICAL AND NON-LINEAR DYNAMICS)

**Full Marks: 40****Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
 Candidates are required to give their answers in their own words as far as practicable.

- 1. Answer any FOUR questions from the following: 4×2=08**
- Using Euler-Lagrange's equation, prove that the shortest distance between two points in a plane is a straight line.
 - A particle is constrained to move on the plane curve $xy = c$ under the influence of gravity only. Obtain its Lagrange's equation of motion.
 - State the basic postulate of special theory of relativity.
 - Define Virtual work and generalized coordinates of a dynamical system.
 - Show that the following transformation is canonical $P = \frac{1}{2}(p^2 + q^2)$, $Q = \tan^{-1}\left(\frac{q}{p}\right)$.
 - Prove that the gain of Kinetic energy of a moving particle is equal to the work done by the particle.

- 2. Answer any FOUR questions from the following: 4×4=16**
- Find the Lagrange's equation of motion for a pendulum in spherical polar coordinates, of length l .
 - Show that the transformation

$$Q = \log(1 + \sqrt{q} \cos p), P = 2\sqrt{q}(1 + \sqrt{q} \cos p) \sin p$$
 is canonical. Find the generating function $G(q, Q)$.
 Then show that
 - the transformation is canonical, and
 - the generating function of this transformation can be put in the form

$$F = -(e^Q - 1)^2 \tan p.$$

2+2
 - Define the degrees of freedom of a dynamical system. Suppose a particle is constraint to move on a circular plane $x^2 + y^2 = a^2$. Then what would be the degree of freedom of the particle?

P.T.O.



(2)

- d) Given a mass-spring system consisting of a mass m and a linear spring of stiffness k hanging from a fixed point. Find the Lagrangian equation of motion for the system.
- e) Write down the Lagrange's equations when the Lagrangian has the following form $L = \dot{q}q - \sqrt{1 - \dot{q}^2}$. Show that the following functional $J = \int_{x_0}^{x_1} \frac{(1+y^2)}{y'^2} dx$ will be extremum if $y = \sinh(c_1x + c_2)$, where c_1, c_2 are arbitrary constant. 1+3
- f) What is the effect of the Coriolis force on a particle falling freely under the action of gravity.

3. Answer any TWO questions from the following: 2×8=16

- a) State Hamilton's principle. Use it to derive the Lagrange's equation of motion for conservative system. 2+6
- b) Prove that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation. Prove that $E = mc^2$, in relativistic mechanics.
- c) Define cyclic coordinates for a dynamical system. For a dynamical system

$$T = \frac{1}{2}(1+2k)\dot{\theta}^2 + 2\dot{\theta}\dot{\phi} + \dot{\phi}^2; V = \frac{1}{2}n^2\{(1+k)\theta^2 + \phi^2\}$$

θ, ϕ are constants, n, k are positive constants. Write down the Lagrange's equation of motion and deduce that

$$\left(\ddot{\theta} - \ddot{\phi}\right) + n^2 \left(\frac{1+k}{k}\right) (\theta - \phi) = 0$$

and if $\theta = \phi, \dot{\theta} = \dot{\phi}$ at $t = 0$ then $\theta = \phi$ for all t . 1+7

- d) Prove that Poisson bracket obeys the distributive law. If X, Y, Z are three dynamical variables, then prove the following:
- (i) $[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0$
- (ii) $[X, Y, Z] = Y[X, Z] + X[Y, Z]$. 2+4+2
