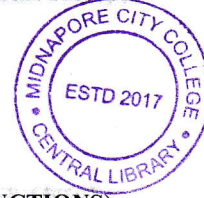


PG CBCS
M.Sc. Semester-I Examination, 2022
DEPARTMENT OF MATHEMATICS
PAPER: MTM 103



(ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS)

Full Marks: 40

Time: 2 Hours

The figures in the right-hand margin indicate full marks.
 Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions from the following: 4×2=08

- a) Write down the hypergeometric series represented by $F(a, b, c; z)$. Prove that $F(1, b, b; z) = \frac{1}{1-z}$.
- b) Define fundamental set of solutions and fundamental matrix for system of ordinary differential equation.
- c) Find all the singularities of the following differential equation and then classify them: $2z(1-z)^2 w'' + 3zw' + (z-2)w = 0$.
- d) Define Green's function of the differential operator L of the non-homogeneous differential equation: $Lu(x) = f(x)$.
- e) Show that $\int_{-1}^1 P_n(z) dz = \begin{cases} 0, & n \neq 0 \\ 2, & n = 0 \end{cases}$ where the symbols have usual meaning.
- f) What are Bessel's functions of order n ? State for what values of n the solutions are independent of Bessel's equation of order n .

2. Answer any FOUR questions from the following: 4×4=16

- a) Show that $(n+1)P_{n+1}(z) + nP_{n-1}(z) = (2n+1)zP_n(z)$.
- b) Using Green's function method, solve the following differential equation $y^{IV}(x) = 1$, subject to boundary conditions $y(0) = y'(0) = y''(1) = y'''(1) = 0$.
- c) If the solution $\varphi_1, \varphi_2, \dots, \varphi_n$ of the linear homogeneous vector differential equation $\frac{dX}{dt} = A(t)X(t)$ be a fundamental solution of the above and $\varphi(t)$ be an arbitrary solution of above. Then prove that φ can be expressed as linear combination of $\varphi_1, \varphi_2, \dots, \varphi_n$ on $[a, b]$.

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(2)

- d) Prove that $\int_{-1}^1 P_m(z)P_n(z)dz = \frac{2}{2n+1}\delta_{mn}$, where δ_{mn} and $P_n(z)$ are the Kroneker delta and Legendre's polynomial respectively.
- e) Deduce Rodrigue's formula for Legendre's polynomial.
- f) Obtain the first five terms in the expansion of the following function f in terms of Legendre's polynomial $f(x) = \begin{cases} 0, & \text{if } -1 < x < 0 \\ x, & \text{if } 0 < x < 1 \end{cases}$

3. Answer any **TWO** questions from the following: 2×8=16

- a) Find the series solution of Legendre's differential equation at the pt. $z = \infty$.
- b) (i) Find the general solution of the equation $2z(1-z)\frac{d^2w}{dz^2} + \frac{dw}{dz} + 4w(z) = 0$ by the method of solution in series about $z=0$ and show that the equation has a polynomial solution which is polynomial in z .
 (ii) Show that $nP_n(z) = zP_n'(z) - P_{n-1}'(z)$, where $P_n(z)$ denotes the Legendre's Polynomial of degree n . 6+2
- c) Find the general solution of the homogeneous system $\frac{dx}{dt} = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Prove that $\frac{d}{dz}(J_0(z)) = -J_1(z)$. 6+2
- d) (i) Define a self-adjoint differential equation with an example.
 (ii) If α and β are the roots of the equation $J_n(z) = 0$ then show that $\int_0^1 J_n(\alpha z)J_n(\beta z)dz = \begin{cases} 0, & \text{if } \alpha \neq \beta \\ \frac{1}{2}[J_n'(z)]^2, & \text{if } \alpha = \beta \end{cases}$ 2+6