*Total pages: 02*

**PG CBCS**

**M.SC. Semester-I Examination, 2022**

**DEPARTMENT OF MATHEMATICS**

PAPER: MTM 103

**(ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS)**

**Full Marks: 40** **Time: 2 Hours**

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The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

**1. Answer any <u>FOUR</u> questions from the following:** **4×2=08**

a) Write down the hypergeometric series represented by F(a, b, c; z). Prove that F(1, b, b; z)=$\frac{1}{1-z}$.

b) Define fundamental set of solutions and fundamental matrix for system of ordinary differential equation.

c) Find all the singularities of the following differential equation and then classify them: $2z(1-z)^2w'' + 3zw' + (z-2)w = 0$.

d) Define Green's function of the differential operator **L** of the non-homogeneous differential equation: $Lu(x) = f(x)$.

e) Show that $\int_{-1}^{1} P_n(z)dz = \begin{cases} 0, & n \neq 0 \\ 2, & n = 0 \end{cases}$ where the symbols have usual meaning.

f) What are Bessel's functions of order n? State for what values of n the solutions are independent of Bessel's equation of order n.

**2. Answer any <u>FOUR</u> questions from the following:** **4×4=16**

a) Show that $(n+1)P_{n+1}(z) + nP_{n-1}(z) = (2n+1)zP_n(Z)$.

b) Using Green's function method, solve the following differential equation $y^{IV}(x) = 1$, subject to boundary conditions $y(0) = y'(0) = y''(1) = y'''(1) = 0$.

c) If the solution $\varphi_1, \varphi_2, \dots\dots, \varphi_n$ of the linear homogeneous vector differential equation $\frac{dX}{dt} = A(t)X(t)$ be a fundamental solution of the above and $\varphi(t)$ be a arbitrary solution of above. Then prove that $\varphi$ can be expressed as linear combination of $\varphi_1, \varphi_2, \dots\dots, \varphi_n$on [a, b].

**P.T.O.**

d) Prove that $\int_{-1}^{1} P_m(z)P_n(z)dz = \frac{2}{2n+1}\delta_{mn}$, where $\delta_{mn}$ and $P_n(z)$ are the Kroneker delta and Legendre's polynomial respectively.

e) Deduce Rodrigue's formula for Legendre's polynomial.

f) Obtain the first five terms in the expansion of the following function f in terms of Legendre's polynomial $f(x) = \begin{cases} 0, & if -1 < x < 0 \\ x, & if\ 0 < x < 1 \end{cases}$

**3. Answer any <u>TWO</u> questions from the following: 2×8=16**

a) Find the series solution of Legendre's differential equation at the pt. $z = \infty$.

b) (i) Find the general solution of the equation $2z(1-z)\frac{d^2w}{dz^2} + \frac{dw}{dz} + 4w(z) = 0$ by the method of solution in series about z=0 and show that the equation has a polynomial solution which is polynomial in z.

(ii) Show that $nP_n(z) = zP_n'(z) - P_{n-1}'(z)$, where $P_n(z)$ denotes the Legendre's Polynomial of degree n. 6+2

c) Find the general solution of the homogeneous system $\frac{dX}{dt} = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Prove that $\frac{d}{dz}\left(J_0(z)\right) = -J_1(Z)$. 6+2

d) (i) Define a self-adjoint differential equation with an example.

(ii) If α and β are the roots of the equation $J_n(z) = 0$ then show that

$$\int_0^1 J_n(\alpha z)J_n(\beta z)dz = \begin{cases} 0, & if\ \alpha \neq \beta \\ \frac{1}{2}[J_n'(z)]^2, & if\ \alpha = \beta \end{cases}$$ 2+6



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