

PG (CBCS)
M.SC. Semester-I Examination, 2022
MATHEMATICS
PAPER: MTM 102
(COMPLEX ANALYSIS)

**Full Marks: 50****Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions from the following:**4×2=08**

- a) Is the function $u = 2xy + 3xy^2 - 2y^3$ harmonic? Justify your answer.
- b) Examine whether the following statement is true or false with proper justification: $\sin(z)$ is bounded in complex plane
- c) Examine whether the following statement is true or false with proper justification: The zeros of $\sin\left(\frac{1}{z}\right)$ are $z = \frac{1}{n\pi}$ ($n \in \mathbb{Z}$) and each zero is isolated.
- d) Suppose that D is a domain and $f \in H(D)$ vanishes throughout any neighborhood of a point in D . Show that $f(z) \equiv 0$ in D .
- e) Suppose ϕ is analytic at z_0 with $\phi(z_0) \neq 0$, g has a pole of order two at z_0 . Prove that:

$$\text{Res}[\phi(z)g(z); z_0] = \phi'(z_0)\text{Res}[(z - z_0)g(z); z_0] + \phi(z_0)\text{Res}[g(z); z_0].$$
- f) Classify the singularity of $f(z) = \sin z$ at ∞ .

2. Answer any FOUR questions from the following:**4×4=16**

- a) Using Cauchy residue theorem, find $\int_C \frac{dz}{z^2 \sin z}$, where C is any circle centered at zero and radius less than π .
- b) Construct a function ϕ which is analytic except at the four distinct points $z_j, j = 1, 2, 3, 4$, where it has the following properties:
 - (i) simple pole at z_1 , (ii) simple zeros at z_2, z_3, z_4
 - (iii) simple pole at infinity, (iv) $\lim_{|z| \rightarrow \infty} z^{-1} \phi(z) = 2$.

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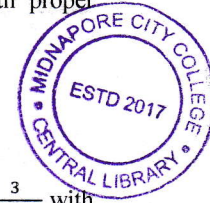
(2)

- c) Examine whether the following statement is true or false with proper justification:

There exists an analytic function in the unit disk Δ such that

$$f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{n}, \text{ for } n \geq 2.$$

- d) Prove that every zeros of an analytic function $f (\neq 0)$ are isolated.
- e) Find the Taylor or Laurent series expansion of the function $f = \frac{3}{z(z-i)}$ with center at $z = -i$ and region of convergence: $1 < |z + i| < 2$.
- f) Show that a power series $\sum_{n \geq 0} a_n z^n$ and the k-times derived series defined by $\sum_{n \geq k} n(n-1)(n-2) \dots (n-k+1) a_n z^{n-k}$ have the same radius of convergence.



3. Answer any TWO questions from the following:

2×8=16

- a) (i) State and prove Jordan's Lemma. 5+3
- (ii) Find the order of the pole of the function $f(z) = \frac{1}{\cos z - \sin z}$ at $z = \frac{\pi}{4}$.
- b) (i) Classify the singularity $z = 0$ of the function $f(z) = \frac{\cos h(z^3) - 1}{z^7}$ in terms of removal, pole and essential singularity. In case $z = 0$ is a pole, specify the order of pole.
- (ii) Evaluate the residue of the function $f(z) = \frac{\cos h(z^3) - 1}{z^7}$ at $z = 0$.
- (iii) Using part-(ii), evaluate $\int_C \frac{\cos h(z^3) - 1}{z^7} dz$, where $C: |z| = 1$ taken in positive direction. 4+2+2

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[3]

- c) (i) Using the method of residue, evaluate: $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$
- (ii) State Cauchy integral formula and evaluate $\int_C \frac{z dz}{z^2-1}$, where $C: |z| = 2$. 4+4
- d) (i) State Rouché's theorem. Use it find the zeros of the polynomial $f(\xi) = \xi^4 - 5\xi + 1$ in the disc $|\xi| < 1$.
- (ii) Find all the Mobius transformation which transforms the half plane $\text{Im}(z) \geq 0$ onto the unit circular disc $|w| \leq 1$. (1+2)+5
