MCC/22/M.SC./SEM.-I/MTM/01

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PG (CBCS)

M.SC. Semester-I Examination, 2022

MATHEMATICS

PAPER: MTM 102

(COMPLEX ANALYSIS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer any FOUR questions from the following: **4×2=08**

a) Is the function $u = 2xy + 3xy^2 - 2y^3$ harmonic? Justify your answer.

b) Examine whether the following statement is true or false with proper justification: $sin\,(z)$ is bounded in complex plane

c) Examine whether the following statement is true or false with proper justification: The zeros of $sin\left(\frac{1}{z}\right)$ are $z = \frac{1}{n\pi}$ $(n \in Z)$ and each zero is isolated.

d) Suppose that D is a domain and $f \in H(D)$ vanishes throughout any neighborhood of a point in D. Show that $f(z) \equiv 0$ in D.

e) Suppose ϕ is analytic at z_0 with $\phi(z_0) \neq 0, g$ has a pole of order two at z_0. Prove that:

$$\text{Res}[\phi(z)g(z); z_0] = \phi'(z_0)\text{Res}[(z - z_0)g(z); z_0] + \phi(z_0)\text{Res}[g(z); z_0].$$

f) Classify the singularity of $f(z) = \sin z$ at ∞.

2. Answer any FOUR questions from the following: **4×4=16**

a) Using Cauchy residue theorem, find $\int_C \frac{dz}{z^2 sin\, z}$, where C is any circle centered at zero and radius less than π.

b) Construct a function ϕ which is analytic except at the four distinct points $z_j, j = 1,2,3,4$, where it has the following properties:

(i) simple pole at z_1, (ii) simple zeros at z_2, z_3, z_4

(iii) simple pole at infinity, (iv) $\lim_{|z|\to\infty} z^{-1}\,\phi(z) = 2$.

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c) Examine whether the following statement is true or false with proper justification:
There exists an analytic function in the unit disk Δ such that

$$f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{n}, for\ n \geq 2.$$

d) Prove that every zeros of an analytic function $f(\not\equiv 0)$ are isolated.

e) Find the Taylor or Laurent series expansion of the function $f = \frac{3}{z(z-i)}$ with center at $z = -i$ and region of convergence: $1 < |z + i| < 2$.

f) Show that a power series $\sum_{n\geq 0} a_n z^n$ and the k-times derived series defined by $\sum_{n\geq k} n(n-1)(n-2)\ldots(n-k+1)a_n z^{n-k}$ have the same radius of convergence.

3. Answer any <u>TWO</u> questions from the following: **2×8=16**

a) (i) State and prove Jordan's Lemma. 5+3
(ii) Find the order of the pole of the function $f(z) = \frac{1}{\cos z - \sin z}$ at $z = \frac{\pi}{4}$.

b) (i) Classify the singularity $z = 0$ of the function $f(z) = \frac{\cos h(z^3)-1}{z^7}$ in terms of removal, pole and essential singularity. In case $z = 0$ is a pole, specify the order of pole.
(ii) Evaluate the residue of the function $f(z) = \frac{\cos h(z^3)-1}{z^7}$ at $z = 0$.
(iii) Using part-(ii), evaluate $\int_C \frac{\cos h(z^3)-1}{z^7} dz$, where $C: |z| = 1$ taken in positive direction. 4+2+2

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c) (i) Using the method of residue, evaluate: $\int_{-\infty}^{\infty} \frac{x^2\, dx}{(x^2+a^2)(x^2+b^2)}$
(ii) State Cauchy integral formula and evaluate $\int_C \frac{z\, dz}{z^2-1}$, where $C: |z| = 2$.
4+4

d) (i) State Rouche's theorem. Use it find the zeros of the polynomial $f(\xi) = \xi^4 - 5\xi + 1$ in the disc $|\xi| < 1$.
(ii) Find all the Mobius transformation which transforms the half plane $Im(z) \geq 0$ onto the unit circular disc $|w| \leq 1$. (1+2)+5

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