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PG (CBCS)

M.SC. Semester-I Examination, 2022

MATHEMATICS

PAPER: MTM 101

(REAL ANALYSIS)

Full Marks: 50 **Time: 2 Hours**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. **Answer any FOUR questions from the following:** **4×2=08**

   a) Let $f(x) = \sin x,\ x \in [a,b]$. Show that f is a function of bounded variation on $[a,b]$.

   b) Show that subtraction of two complex measurable set X is measurable.

   c) Define σ −algebra with an example.

   d) Show that every constant function is measurable

   e) If α is continuous and β is of bounded variation on $[a,b]$, show that $\int_a^b \alpha d\beta$ exists.

   f) If $s_1, s_2 \in L_0^+$ then prove that $s_1 + s_2 \in L_0^+$.

2. **Answer any FOUR questions from the following:** **4×4=16**

   a) Show that the function $f(x) = x^2 sin\frac{\pi}{x^2}, x \in (0,1]$ and $f(0) = 0$ is not a function of bounded function [0, 1].

   b) Check whether the function $f(x) = |5x - 1| + |x|$ on[0, 3] is a function of bounded variation or not. If so, also find the variation function of f on [0, 3].

c) Let $[a, b]$ be a closed and bounded interval. Let $f: [a, b] \to \mathbb{R}$ be continuous functions on $[a, b]$. Show that f is a measurable function.

d) Let $f \in L^+$ and $E \in S$ such that $\int_E f d\mu = 0$. Prove that $f(x) = 0$ for a.e. $\mu(x) \in E$.

e) Let $f_n: X \to \mathbb{R}^*$ be measurable for $n = 1, 2, 3, \ldots\ldots$ Then show that $\lim inf_{n\to\infty} f_n$ and $inf_{n\to\infty} f_n$ are measurable functions on X.

f) If $f_n: X \to [0, \infty]$ is measurable for $n = 1, 2, 3, \ldots\ldots$ and $f(x) = \sum_{n=1}^{\infty} f_n(x),\ x \in X$, then show that $\int f d\mu = \sum_{n=1}^{\infty} \int f_n d\mu$.

3. Answer any <u>TWO</u> questions from the following: **2×8=16**

a) Let $f: [a, b] \to \mathbb{R}$ and $\alpha: [a, b] \to \mathbb{R}$ be two functions. If f is monotonic on $[a, b]$ and α is monotone and continuous on $[a, b]$ then show that f is Riemann-Stieltjes integrable with respect to α on $[a, b]$. Evaluate $\int_0^4 (x - [x]) dx^2$.

b) (i) Let $f(x) = \frac{1}{x^p}\ if\ 0 < x \leq 1$ and $f(0) = 0$. Find necessary and sufficient condition on p such that $f \in L^1[0, 1]$. Compute $\int_0^1 f(x)\lambda(x) dx$ in that case.

(ii) Evaluate the following:

$\int_{-1}^{3} 7sinx\ d(x - 3[x])$. 6+2

c) (i) Suppose $f: X \to [0, \infty]$ is measurable and $\varphi(E) = \int_E f d\mu$ for every measurable set E in X. Show that φ is a measure and $\int g d\varphi = \int gf\ d\mu$ for every measurable function g on X with range in $[0, \infty]$.

(ii) Show that the Cantor set is a null set. 6+2

d) State and prove monotone convergence theorem.

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