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বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

BCA

2nd Semester Examination 2022

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

PAPER—1203

Full Marks : 100

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group – A

Answer any *four* questions. 4×15

1. (a) (i) Find the nature of the roots of the equation

$$x^6 - 3x^2 - x + 1 = 0$$

by Descartes' rule of signs.

(b) Solve the equation $x^3 - 3x^2 + 4 = 0$ whose two roots are equal.

(c) If α, β, γ be the roots of $x^3 + qx + r = 0$, find the equation whose roots are $\frac{\alpha\beta}{\gamma}, \frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}$. 5+5+5

2. (a) Prove that $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix} = 0$

(b) Solve by cramer's rule

$x + y - z = 6$

$2x - 3y + z = -1$

$3x - 4y + 2z = -1$

3. (a) Solve the system of equations by matrix method

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$

(b) If A, B are any two events then show that,

$P(AUB) = P(A) + P(B) - P(A \cap B)$

(c) Check the functions bijective or not, where $f(x) = |x| \mid x$

5+5+5

4. (a) Determine k so that (1, 3, 1), (2, k, 0) and (0, 4, 1) are linearly dependent.

(b) Find the ligen values of $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

(c) If 3A − B = $\begin{pmatrix} -2 & 6 & 1 \\ -3 & -4 & 7 \\ 3 & -17 & 5 \end{pmatrix}$, A + 2B = $\begin{pmatrix} 4 & -5 & -2 \\ 6 & 8 & -7 \\ 1 & 34 & -10 \end{pmatrix}$ find A and B.

5+5+5

5. (a) Show that the mapping f : z → z defined by f(x) = 3x, $x \in z$ is injective but not surjective.

(b) If A = $\begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$, show that $A^2 - 10A + 16I_3 = 0$. Hence Obtain A^{-1}.

(c) Solve by matrix method

x + y + z = 1

2x + y + 2z = 1

x + 2y + 3z = 0.

5+5+5

6. (a) If $y = \left(x^2 - 1\right)^n$, then show that

$$\left(x^2 - 1\right)y_{n+2} + 2xy_{n+1} - n\left(n+1\right)y_n = 0$$

(b) If $y = x^{2n}$ where n is a positive integer, show that

$y_n = 2^n \{1.3.5......(2n-1)\}x^n$.

(c) Verify Rolle's theorem for the function

$f(x) = x^3(x - 1)^2$ in the interval $0 \le x \le 1$. 5+5+5

7. (a) Use Mean value theorem to prove the inequality

$$0 < \frac{1}{x}\log\frac{e^x - 1}{x} < 1$$

(b) Evaluate $\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$

(c) Evaluate $\int_0^{\pi 2} \frac{dx}{5+3\cos x}$. 5+5+5

8. (a) If $u = z\tan^{-1}\frac{y}{z}$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

(b) Evaluate $\lim_{x \to \infty}\left[\frac{n!}{n^n}\right]^{1/n}$

(c) State Euler's on homogeneous function and verify it on $u = ax^2 + 2hxy$ by^2.

Group – B

Answer any *one* question. 1×10

9. (a) A box contains 12 blue and 8 black pens. If 4 pens are randomly taken from the box, find the probability that

(i) no black pen is taken

(ii) at least two blue pens are taken.

(b) A and B are two events associated with the same experiment E and $P(A + B) = \frac{5}{6}$, $P(AB) = \frac{1}{3}$ and $P(\bar{A}) = \frac{2}{3}$. Find P(A), P(B) and $P(A\bar{B})$.

10. (a) A box contain 3 white, 5 red and 4 black balls. One ball is drawn at random and is replaced before the next draw. Find the expection and variance of the number of white balls iff if the experiment be repeated 80 times in succession.

(b) Explain the terms 'skewness' and 'Kurtosis' with their different measrues.

(Internal Assessment : 30)