

2022

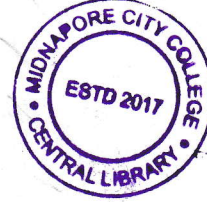
BCA 1st Semester (CBCS) Examination

Mathematics

PAPER — BCAGE — 1T

Full Marks : 80

Time : 3 hours



The figures in the right-hand margin indicate marks.

Candidates are required to give their answers  
in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer from *any one* Section as directed.

SECTION—A

Basic Mathematics (GE 1.1)

Answer from *all* the Groups as directed.

GROUP—A

1. Answer *any ten* questions : 2×10=20

(a) Express  $\log_{10} 1 = 0$  in exponential form.

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$$10^{\log_{10} 1} = 10^0$$
$$\therefore 1 = 10^0$$

(Turn Over)

(2)

- (b) What is logarithmic function?
- (c) If  $U = \{a, b, c, d, e, f\}$ ;  $A = \{a, b, c\}$ ;  $B = \{c, d, e, f\}$ ;  $C = \{c, d, e\}$ ; then find  $(A \cap B) \cup (A \cap C)$ .
- (d) Define group.
- (e) Define a scalar matrix with an example.
- (f) Define an equivalence relation.
- (g) Define permutation and combination.
- (h) Find the value of  $\log_2 2 + \log_2 2^2 + \log_2 2^3 + \dots + \log_2 2^n$ .
- (i) What is the value of  $\lim_{x \rightarrow 0} \frac{\sin x + \log(1-x)}{x^2}$ ?
- (ii) 0
- (iii) 1
- (iv)  $-\frac{1}{2}$
- (v)  $\frac{1}{2}$
- (j) What is cartesian product?
- (k) Define symmetric matrix.
- (l) Find the value of  ${}^5C_3$ .
- (m) Write inverse of a matrix.
- (n) Define reflexive relation.
- (o) What is Cramer's rule?

Handwritten calculations for Cramer's rule:

$$\frac{15}{5 \times 4 \times 12} = \frac{15}{240} = \frac{1}{16}$$

$$\frac{12}{5 \times 4 \times 15} = \frac{12}{300} = \frac{2}{50} = \frac{1}{25}$$

$$\frac{12}{5 \times 4 \times 12} = \frac{12}{240} = \frac{1}{20}$$

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$$\frac{12}{5 \times 4 \times 12} = \frac{12}{240} = \frac{1}{20}$$

(3)  
GROUP-B

2. Answer any six questions : 5×6=30

- (a) Write down the laws of indices.
- (b) Prove that 7, 11, 15, 19 and 23 is in AP.
- (c) Define diagonal matrix with example.
- (d) Find the conjugate of  $\sqrt{-3} + 4i^2$ .
- (e) Let  $V$  be the vector space of  $n \times n$  matrices over the field of real numbers and  $W_1$  and  $W_2$  be subspaces of symmetric and skew symmetric matrices of order  $n$  respectively. Show that  $V = W_1 \oplus W_2$
- (f) Verify the Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix}$ .
- (g) Let a relation  $R$  be defined by  $R = \{(4, 5); (1, 4); (4, 6); (7, 6); (3, 7)\}$ . Then what is  $R^{-1} \circ R$ ?
- (h) Show that  $\begin{bmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{bmatrix} = b^3 + a^3$ .

(Turn Over)

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GROUP—C

Answer any three questions :

3. (a) Prove that there exists a basis for each finitely generated vector space.

- (b) Find the value of  $x$  and the value of  $y$  in the following equation, given further that  $x \in R, y \in R$  :

$$5+5=10$$

$$(x + iy)(2 + i) = (3 - i)$$

4. Compute the dimension and find bases for the following linear spaces :

- (a) Real anti-symmetric  $4 \times 4$  matrices.

- (b) Quartic polynomials  $p$  with the property that  $p(2) = 0$  and  $p(3) = 0$ . 5+5=10

5. Call a subset  $S$  of a vector space  $V$  a spanning set if  $\text{Span}(S) = V$ . Suppose that  $T : V \rightarrow W$  is a linear map of vector spaces.

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- (a) Prove that a linear map  $T$  is 1-1 if and only if  $T$  sends linearly independent sets to linearly independent sets.

- (b) Prove that  $T$  is onto if and only if  $T$  sends spanning sets to spanning sets. 5+5=10

6. (a) Find a real  $2 \times 2$  matrix  $A$  (other than  $A = I$ ) such that  $A^5 = I$

- (b) Find an example of  $2 \times 2$  matrices  $A$  and  $B$  so that  $AB = 0$  but  $BA \neq 0$ . 5+5=10

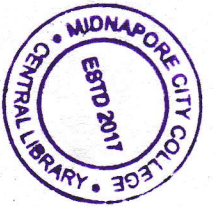
7. (a) Solve the equations by Cramer's rule  
 $3x - y + 2z = 13,$   $2x + y - z = 3$  and  
 $x + 3y - 5z = -8.$

- (b) If  $\log\left(\frac{a-b}{5}\right) = \frac{1}{2}(\log a + \log b)$ , then show that  $a^2 + b^2 = 27ab$ . 5+5=10

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(Turn Over)

$\log \frac{a-b}{5} = \log(ab)^{1/2}$   
 $\log \frac{a-b}{5} = \log \sqrt{ab}$   
 $\frac{(a-b)}{5} = \sqrt{ab}$   
 $a^2 - 2ab + b^2 = 25ab$



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**SECTION—B**

**Numerical Method (GE 1.2)**

Answer from **all** the Groups as directed.

**GROUP—A**

1. Answer **any ten** questions :  $2 \times 10 = 20$

- (a) If  $\pi = 3.14$  is used in place of 3.14156, then find the absolute error and relative error.
- (b) Round off the following numbers correct up to three significant figures :  
0.9754, 0.05681.
- (c) What is interpolation polynomial?
- (d) What is forward difference operator?
- (e) Prove that  $E \equiv 1 + \Delta$ .
- (f) Write the advantage of Regula-Falsi Method.
- (g) Distinguish between round off error and truncation error.
- (h) Which interpolation formula is suitable if argument of  $f(x)$  is near the end of the table?

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- (i) Find the function for  $f(2) = 7, f(5) = 34, f(7) = 62$  by Lagrange's formula.
- (j) State the condition for the existence of root of an equation within given range  $[a, b]$ .
- (k) Distinguish between Simpson's 1/3rd and trapezoidal integration formula.
- (l) Write the error term of trapezoidal rule.
- (m) What is the advantage of Gauss-Seidal method over Gauss-Jacobi method?
- (n) What is relative percentage error?
- (o) What is extrapolation?

**GROUP—B**

2. Answer **any six** questions :  $5 \times 6 = 30$

- (a) Write the sufficient condition of convergence of the method of iteration process for solving the system of linear equations.
- (b) Write the geometrical interpretation of trapezoidal formula.
- (c) Establish the relation between forward difference and backward difference.

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(Turn Over)



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- (d) Compute  $y(0.02)$  by Euler method for the problem  $\frac{dy}{dx} = x^3 + y$ ,  $y(0) = 1$ , taking  $h = 0.01$ .
- (e) Evaluate  $\int_0^1 \frac{1-x}{1+x} dx$  by using trapezoidal rule with 10 sub-intervals.
- (f) Explain bisection method for algebraic equation.
- (g) Find the condition of convergence of the Newton-Raphson method.
- (h) Write the advantages and disadvantages of Lagrangian interpolation formula.

**GROUP—C**

3. Answer any three questions :

- (a) Find a real root of the equation  $f(x) = x^3 + x^2 + x + 7 = 0$  by using bisection method.
- (b) Establish Regula-Falsi method. 5+5=10
4. Establish Newton forward difference interpolation formula. 10

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(Continued)

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5. Calculate  $f(1.30)$  from the given table :

x	0.0	1.2	2.4	3.7
f(x)	3.41	2.68	1.37	-1.18

Using Lagrangian interpolation formula. 10

6. Evaluate  $f(1.2)$  and  $f(3.8)$  using following table :

x	0	1	2	3	4
f(x)	1	1.5	2.2	3.1	4.3

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7. (a) Evaluate  $\int_0^1 \frac{x}{1+x} dx$  by using Simpson's

1/3rd rule, taking six intervals.

(b) For the given set of values of  $y = f(x)$ ,

x	1	3	5	7	9
f(x)	8	12	21	36	62

form the backward difference table and find the values of  $\nabla f(3)$ ,  $\nabla^2 f(7)$  and  $\nabla^3 f(9)$ .

5+5=10

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